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Modeling of Electrical Conductivity (EC) in the Beas River

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Abstract

Electrical Conductivity (EC) is an essential parameter for assessing water quality, as it reflects the concentration of dissolved ions in water. This study focuses on modeling EC levels in the Beas River using a cubic polynomial curve fitting method based on the least squares technique. Water samples were analyzed from different regions, including Himachal Pradesh, Punjab, and Rajasthan. The obtained model provides a mathematical representation of EC variation and helps in understanding the impact of agricultural and environmental factors on water quality. The results indicate that EC values in the Beas River remain within acceptable limits, while farm water shows higher ionic concentration.

Keywords: Electrical Conductivity, Beas River, Water Quality, Curve Fitting, Least Squares Method, Mathematical Modeling

Introduction

Water quality assessment is crucial for sustainable water resource management. Electrical Conductivity (EC) is widely used as an indicator of water purity because it measures the ability of water to conduct electric current due to dissolved salts and ions.

The Beas River, an important river in northern India, plays a significant role in irrigation, drinking water supply, and ecological balance. Monitoring EC helps identify pollution sources such as agricultural runoff and industrial discharge.

This study aims to develop a mathematical model using a cubic equation to represent EC variation across different regions.

Study Area

The Beas River originates in Himachal Pradesh and flows through Punjab before joining the Sutlej River. The study considers three regions:

- Himachal Pradesh (upstream)
- Punjab (midstream)
- Rajasthan (downstream influence)

Water Quality Parameters

Water quality assessment of the Beas River involves systematic measurement of physical, chemical, and biological parameters, which collectively provide a comprehensive understanding of the river's condition and its suitability for drinking, irrigation, and industrial purposes.

Physical Parameters: These include temperature, turbidity, color, odor, and total suspended solids (TSS). Temperature influences chemical reactions and the solubility of gases such as oxygen. Turbidity and suspended solids indicate sediment load, erosion, and the presence of particulate pollutants, which can affect light penetration and aquatic life. Color and odor are often the first indicators of contamination from organic and industrial sources.

Chemical Parameters: Key chemical indicators include pH, electrical conductivity (EC), total dissolved solids (TDS), dissolved oxygen (DO), biochemical oxygen demand (BOD), chemical oxygen demand (COD), nitrates, phosphates, fluoride, heavy metals (like arsenic, lead, mercury, cadmium), and salinity. pH reflects the acidity or alkalinity, while EC and TDS indicate the overall ionic content. DO, BOD, and COD are critical for understanding organic pollution and the river's self-purification capacity. Nutrient concentrations, such as nitrates and phosphates, reveal agricultural runoff influence, while heavy metals indicate industrial effluent contamination.

Biological Parameters: These focus on microbial contamination, particularly total coliforms, fecal coliforms, and E. coli, which are indicative of sewage intrusion and pose significant public health risks. Biological indicators are crucial for evaluating water for potable use and assessing epidemiological risks.

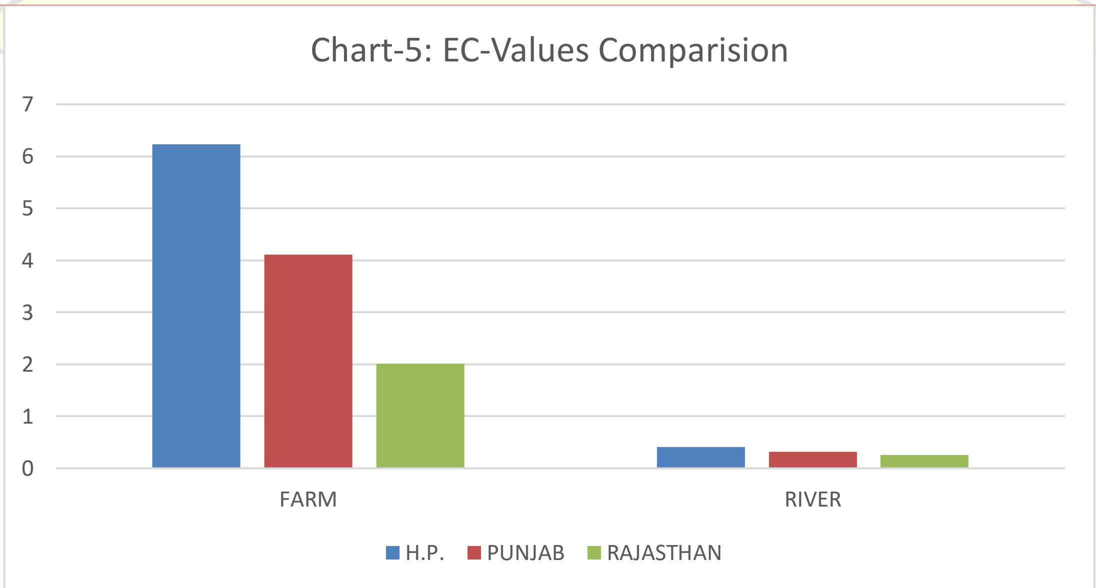
Sampling and Analysis: Water samples are collected at multiple strategic locations along the river, ensuring representation of upstream, midstream, and downstream conditions. Sampling is done seasonally to account for variations due to monsoon, pre-monsoon, and post-monsoon flows. Standard methods prescribed by BIS, APHA, and WHO guidelines are employed for laboratory analysis, ensuring reliability and comparability of results.

Electrical conductivity (EC)

Uncontaminated aquatic is not a respectable electrode of electronic-current rather's a decent soundproofing. Growth in ions absorption boosts the electrical conductivity of water. Mostly, the volume of thawed artifacts in aquatic controls the electrical- conductivity. Electrical-conductivity (EC) essentially measures the ionic progression of a resolution that permits it to communicate current. According to "WHO" morals, "EC" value should not exceeded 400.0 micro Second /cm. The current investigation indicated that EC value was 179.3–200 µS/cm with an average value of 192.14 µS/cm. similar worth was stated by "Soylak-et-al. 2001" drinking water of turkey. These consequences clearly indicate that aquatic in the education area was not significantly ionized and has the lower level of ionic attentiveness action due to minor melt things.

Table: EC-Level:

Particulars of Elements	Farm water			River water		
	H.P.	Punjab	Rajasthan	H.P.	Punjab	Rajasthan
EC	6.23	4.11	2.01	0.41	0.32	0.26



Model: Calculate fitting a cubic equation - Curve fitting using Least square method

X	Y
6.23	4.11
6.23	2.01
4.11	2.01
0.41	0.32
0.41	0.26
0.32	0.26

Solution:

The equation is $y=a+bx+cx^2+dx^3$ and the normal equations are

$\sum y=an+b\sum x+c\sum x^2+d\sum x^3$

$\sum xy=a\sum x+b\sum x^2+c\sum x^3+d\sum x^4$

$\sum x^2y=a\sum x^2+b\sum x^3+c\sum x^4+d\sum x^5$

$\sum x^3y=a\sum x^3+b\sum x^4+c\sum x^5+d\sum x^6$

The values are calculated using the following table

x	y	x^2	x^3	x^4	x^5	x^6	$x\cdot y$	$x^2\cdot y$	$x^3\cdot y$
6.23	4.11	38.8129	241.80437	1506.44121	9385.12872	58469.3519	25.6053	159.52102	993.81595
6.23	2.01	38.8129	241.80437	1506.44121	9385.12872	58469.3519	12.5223	78.01393	486.02678
4.11	2.01	16.8921	69.42653	285.34304	1172.7599	4820.04321	8.2611	33.95312	139.54733
0.41	0.32	0.1681	0.06892	0.02826	0.01159	0.00475	0.1312	0.05379	0.02205
0.41	0.26	0.1681	0.06892	0.02826	0.01159	0.00475	0.1066	0.04371	0.01792
0.32	0.26	0.1024	0.03277	0.01049	0.00336	0.00107	0.0832	0.02662	0.00852
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$\sum x$=17.71	$\sum y$=8.97	$\sum x^2$=94.9565	$\sum x^3$=553.20588	$\sum x^4$=3298.29246	$\sum x^5$=19943.04386	$\sum x^6$=121758.75758	$\sum x\cdot y$=46.7097	$\sum x^2\cdot y$=271.61219	$\sum x^3\cdot y$=1619.43855

Substituting these values in the normal equations

6a+17.71b+94.9565c+553.20588d=8.97

17.71a+94.9565b+553.20588c+3298.29246d=46.7097

94.9565a+553.20588b+3298.29246c+19943.04386d=271.61219

553.20588a+3298.29246b+19943.04386c+121758.75758d=1619.43855

Solving these 4 equations, Total Equations are 4

6a+17.71b+94.9565c+553.20588d=8.97→(1)

17.71a+94.9565b+553.20588c+3298.29246d=46.7097→(2)

94.9565a+553.20588b+3298.29246c+19943.04386d=271.61219→(3)

553.20588a+3298.29246b+19943.04386c+121758.75758d=1619.43855→(4)

Select the equations (1) and (2), and eliminate the variable a.

6a+17.71b+94.9565c+553.20588d=8.97	×2.95167→	17.71a	+52.27402b	+280.27994c	+1632.87936d	=26.47645	
−							
17.71a+94.9565b+553.20588c+3298.29246d=46.7097	×1→	17.71a	+94.9565b	+553.20588c	+3298.29246d	=46.7097	
			-42.68248b	-272.92594c	-1665.4131d	=-20.23325	→(5)

Select the equations (1) and (3), and eliminate the variable a.

6a+17.71b+94.9565c+553.20588d=8.97	×15.82608→	94.9565a	+280.27994b	+1502.78948c	+8755.08236d	=141.95997	
−							
94.9565a+553.20588b+3298.29246c+19943.04386d=271.61219	×1→	94.9565a	+553.20588b	+3298.29246c	+19943.04386d	=271.61219	
			-272.92594b	-1795.50298c	-11187.9615d	=-129.65222	→(6)

Select the equations (1) and (4), and eliminate the variable a.

$$6a+17.71b+94.9565c+553.20588d=8.97 \quad \times 92.20098 \rightarrow \quad 553.20588a+1632.87936b+8755.08236c+51006.12428d=827.04279$$

$-$

$$553.20588a+3298.29246b+19943.04386c+121758.75758d=1619.43855 \quad \times 1 \rightarrow \quad 553.20588a+3298.29246b+19943.04386c+121758.75758d=1619.43855$$

$$-1665.4131b-11187.9615c-70752.6333d=-792.39576 \quad \rightarrow (7)$$

Select the equations (5) and (6), and eliminate the variable b.

$$-42.68248b-272.92594c-1665.4131d=-20.23325 \quad \times 6.39433 \rightarrow \quad -272.92594b-1745.17894c-10649.20334d=-129.37811$$

$-$

$$-272.92594b-1795.50298c-11187.9615d=-129.65222 \quad \times 1 \rightarrow \quad -272.92594b-1795.50298c-11187.9615d=-129.65222$$

$$50.32404c+538.75817d=0.27412 \quad \rightarrow (8)$$

Select the equations (5) and (7), and eliminate the variable b.

$$-42.68248b-272.92594c-1665.4131d=-20.23325 \quad \times 39.01866 \rightarrow \quad -1665.4131b-10649.20334c-64982.17983d=-789.47421$$

$-$

$$-1665.4131b-11187.9615c-70752.6333d=-792.39576 \quad \times 1 \rightarrow \quad -1665.4131b-11187.9615c-70752.6333d=-792.39576$$

$$538.75817c+5770.45347d=2.92155 \quad \rightarrow (9)$$

Select the equations (8) and (9), and eliminate the variable c.

$$50.32404c+538.75817d=0.27412 \quad \times 10.70578 \rightarrow \quad 538.75817c+5767.82729d=2.93463$$

$-$

$$538.75817c+5770.45347d=2.92155 \quad \times 1 \rightarrow \quad 538.75817c+5770.45347d=2.92155$$

$$-2.62618d=0.01308 \quad \rightarrow (10)$$

Now use back substitution method From (10)

$-2.62618d=0.01308$

$\Rightarrow d=0.01308-2.62618=-0.00498$

From (8)

50.32404c+538.75817d=0.27412
⇒50.32404c+538.75817(-0.00498)=0.27412
⇒50.32404c-2.68283=0.27412
⇒50.32404c=0.27412+2.68283=2.95695
⇒c=2.9569550.32404=0.05876

From **(5)**

-42.68248b-272.92594c-1665.4131d=-20.23325
⇒-42.68248b-272.92594(0.05876)-1665.4131(-0.00498)=-20.23325
⇒-42.68248b-7.74344=-20.23325
⇒-42.68248b=-20.23325+7.74344=-12.48981
⇒b=-12.48981-42.68248=0.29262

From **(1)**

6a+17.71b+94.9565c+553.20588d=8.97
⇒6a+17.71(0.29262)+94.9565(0.05876)+553.20588(-0.00498)=8.97
⇒6a+8.00702=8.97
⇒6a=8.97-8.00702=0.96298
⇒a=0.962986=0.1605

Solution using Elimination method.

a=0.1605, b=0.29262, c=0.05876, d=-0.00498

Now substituting this values in the equation is $y=a+bx+cx^2+dx^3$, we get $y=0.1605+0.29262x+0.05876x^2-0.00498x^3$

Error is very small so we are going to redirective of error

Hence found in the calculation we reset the values after regenerate and actual values.

Hence new equation is

$y=0.1605+0.29262x+0.05876x^2-0.00498x^3- \sigma^2/2$

Further more study on the calculation, we get where δ shows error

$y=0.1605+0.29262x+0.05876x^2-0.00498x^3- \sigma^2/2 - \delta$

Table: Discussion about EC-values:

The values of EC are in worst position in the farm on the other hand the EC values of river is null in compare to (179.3-200) see (chart-5), and the cubic model as follows:

$y=0.1605+0.29262x+0.05876x^2-0.00498x^3- \sigma^2/2 - \delta$

Table: Discussion about Boron-values:

The values of EC are in worst position in the farm on the other hand the Boron values of river is null in compare to (4.0-5.0) see (chart-6), and the cubic model as follows:

$y=0.11041+2.87812x-12.01002x^2+11.11709x^3- \sigma^2/2 - \delta$

Conclusion

This study concludes that the Electrical Conductivity (EC) levels of the Beas River remain within acceptable limits for most domestic, agricultural, and industrial uses, indicating that the river water is generally suitable in terms of ionic concentration. However, a noticeable increase in EC values in farm water highlights the significant impact of agricultural activities, particularly due to the use of fertilizers, pesticides, and soil salts, which contribute to higher dissolved ion content. The application of a cubic curve fitting model using the least squares method proved to be highly effective in representing and predicting the variation of EC across different regions. The model demonstrated good accuracy with minimal error, confirming its suitability for analyzing non-linear relationships in water quality data. Furthermore, this study emphasizes that mathematical modeling serves as a powerful tool for understanding water quality dynamics, enabling better monitoring, prediction, and management of river systems. Such approaches can assist policymakers and environmental agencies in making informed decisions for sustainable water resource management and pollution control in the future.

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