Study of Algorithms to Calculate Real Root of Transcendental Equations

Arvind Panwar, Department of computer science, R D engineering College, Ghaziabad
Corresponding author-arvind.panwar@gmail.com

Abstract

This study deals with a comparative analysis of algorithms for solving transcendental equations. It includes algorithms for RF-EXP, RF-Halley, Arc-sine, Tanh and RF-ArcTanh methods to check the accuracy, number of iterations, and errors for the solution of the trigonometric, logarithmic, and exponential equations. Several numerical examples are presented to illustrate the algorithms' efficacy which are programmed in MATLAB. The findings indicate that Arc-sine and RF-ArcTanh, RF-Halley, and Tanh are in favour of exponential equations, trigonometric equations, and logarithmic equations respectively.

Keywords: RF-EXP, RF-Halley, Arc-sine, Tanh and RF-ArcTanh

1. INTRODUCTION

Many different academic and professional sectors commonly use equations that don't follow a linear trend. These topics include the use of numerical methods to generate an accurate estimate of the solution for these equations. This is because it could be difficult or even impossible to solve certain equations analytically. The most frequently chosen subset of numerical strategies for solving nonlinear equations has been iterative methods. An introduction to various iterative methods for the solution of nonlinear equations will be given within the context of this discussion. These methods are based on the idea of estimating the solution initially, followed by several iterative rounds of refinement, carried out until the necessary level of precision is attained. In many disciplines, including physics, engineering, and mathematics, nonlinear equations are crucial. Depending on the numerical approach's convergence order, it will reach the exact solution. It is common knowledge that many issues that emerge in a variety of pure and applied science domains can be expressed in terms of nonlinear equations of the form f(x) = 0. [1] The traditional Newton-Raphson method (NM) is the most well-known of these techniques [2].

$f'(x_n)$For a given initial selection x_0, we get the approximate solution x_{n+1} using the iterative formula

$$- \frac{2f(x_n)f'(x_n)}{2[f'(x_n)]^2-(x_n)f''(x_n)}$$

This is referred to as the cubic convergence of Halley's approach [3].

2. LITERATIVE METHODS

Arc-Sine Algorithm

A hybrid algorithm is presented by [4], the iterative formula used in the trigonometrical method is as follows [5]. For n = 0, 1, 2,…

$$x_{n+1} = x_n \frac{-f(x_n)}{[1 + \arcsin(\quad x_{n}f \quad x_n)]} \qquad (3)$$

In regula-falsi method, we take two initial guesses say a and b such that the product of f(a) and f(b)

should be less than zero. The approximate root is calculated by discovering the point of intersection of the straight line

$$x_r = a - \frac{f(a)(b-a)}{f(b)-f(a)} \qquad (4)$$

the coordinates (a, f(a)) and (b, f(b)) with the x-axis. Hence, the estimated root can be calculated by the formula,

ALGORITHM

1. $i = 0$
2. while $i != n$ do
3. $i = i + 1;$
4. $x_{rf} = a - \frac{f(a)(b-a)}{f(b)-f(a)}$
5. x_{n+1} $[1 + \arcsin($ $= x_{rf}$ $)]$

$\bar{f}\,(x_r F)$ $x_{rF} f \quad x_{rF})$

6. if $|a - xi| \le$ eps then
7. return xi, $f(x_i)$ break;
8. else if $f(x_i) \times f(a) < 0$ then $b = x_i$
9. else $a = x_i$
10. end (if)
11. end (while)

Regula Falsi-Exponential Algorithm(RF-EXP)

In this section, a hybrid algorithm using the regula-falsi method and the exponential method (RF-EXP) by [6] is presented. The regula-falsi method guarantees the existence of the root, while the exponential method gives faster convergence. The iterative formula used in the exponential method is as follows [7]

$$x_{n+1} = x_n \exp\left(\frac{-f(x_n)}{x_n f'(x_n)}\right) \quad (5)$$

In the regula-falsi method, we take two initial guesses, say a and b, such that $f(a)f(b) < 0$. The approximate root is calculated by finding the point of intersection of the straight line joining the points

Regula Falsi-Halley Algorithm(RF-Halley)

The Halley's method is invented by Edmond Halley. In this method, we need one initial approximation as in Newton's method with a continuous second derivative, and this method produces a sequence of approximations to the root. We compute the sequence of iterations using the Halley's method formula by [6]

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2[f'(x_n)]^2 - (x_n)f''(x_n)} \quad (6)$$

with an initial approximation x_0.In this algorithm, we have:

- the input: the function f(x), the interval [a, b] where the exact root lies, the absolute error eps, the number of iterations n;
- the output: the approximate root x, the function value f(x). The steps of the algorithm are as follows

ALGORITHM

1. $i = 0$
2. while i! = n do
3. $i = i + 1;$
4. $x_{rf} = a - \frac{f(a)(b-a)}{f(b)-f(a)}$
5. $x_{n+1} = x_{rf} - \frac{2f(x_{rF})f'(x_{rF})}{2[f'(x_{rF})]^2 - (x_{rF})f''(x_{rF})}$
6. if $|a - xi| \le$ eps then
7. return xi, $f(x_i)$ break;
8. else if $f(x_i) \times f(a) < 0$ then $b = x_i$
9. else $a = x_i$
10. end (if)
11. end (while)

Tangent Hyperbolic Algorithm(Tanh)

The hyperbolic tangent iterative formula using tanh [8] is proposed as

$$x_{n+1} = x_n \left[1 + \tanh\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)\right] \quad (7)$$

By expanding this iterative formula, one can obtain the standard Newton-Raphson method as in the first two terms.

Using the usual expansion of tanh as

$$\tanh(x) = x - \frac{x^3}{3!} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots, \quad |x| < 1$$

ALGORITHM

1. $i = 0$
2. while $i! = n$ do
3. $i = i + 1;$
4. $x_{n+1} = x_n \left[1 + \tanh\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)\right]$
5. if $|a - x_i| \leq eps$ then
6. return x_i, $f(x_i)$ break;
7. else if $f(x_i) \times f(a) < 0$ then $b = x_i$
8. else $a = x_i$
9. end (if)
10. end (while)

Regula-Falsi arc tangent hyperbolic algorithm (RF-Arctanh)

Here, we proposed a new algorithm which is the combination of arc-tanh and regula falsi method which is the combination of regula falsi method and arc-hyperbolic tangent function given as follows

$$x_{n+1} = x_n \left[1 + \operatorname{arctanh}\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)\right] \quad (8)$$

The approximate root is calculated by discovering the point of intersection of the straight line combining

$$x_r = a - \frac{f(a)(b-a)}{f(b) - f(a)}$$

the coordinates $(a, f(a))$ and $(b, f(b))$ with the x-axis. Hence, the estimated root can be calculated by the formula,

ALGORITHM

1. $i = 0$
2. while $i! = n$ do
3. $i = i + 1;$
4. $x_{rf} = a - \frac{f(a)(b-a)}{f(b)-f(a)}$
5. $x_{n+1} = x_{rf} \left[1 + \operatorname{arctanh}\left(\frac{-f(x_{rF})}{x_{rF} f'(x_{rF})}\right)\right]$
6. if $|a - x_i| \leq eps$ then
7. return x_i, $f(x_i)$ break;
8. else if $f(x_i) \times f(a) < 0$ then $b = x_i$
9. else $a = x_i$
10. end (if)
11. end (while)

3. Numerical Experiment

In this example, we present a comparison between various existing methods and the proposed algorithms to show the efficiency and simplicity of the proposed algorithms in computation of root and comparisons are taken into account to confirm which algorithm is more efficient for different types of equations. Here,

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the three types of equations: A, B, and C, which are exponential, trigonometric, and logarithmic respectively with initial approximations a and b are taken.

Table 1: Exact root of equations

Name	Functions	Exact Root	Initial Approximations
A	$e^x - 3x - 2$	2.1253911988	a=2,b=3
B	$x - \cos x$	0.7390851332	a=0,b=1
C	$\log x$	1.00000000	a=0.5,b=2

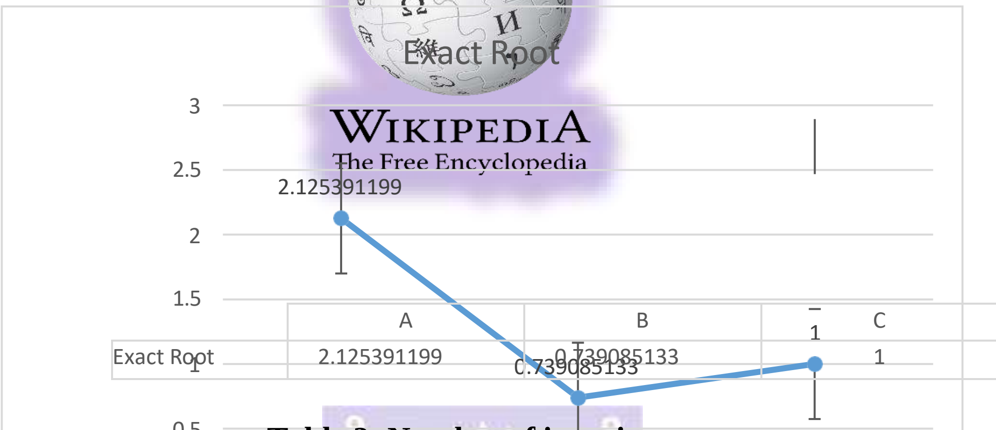
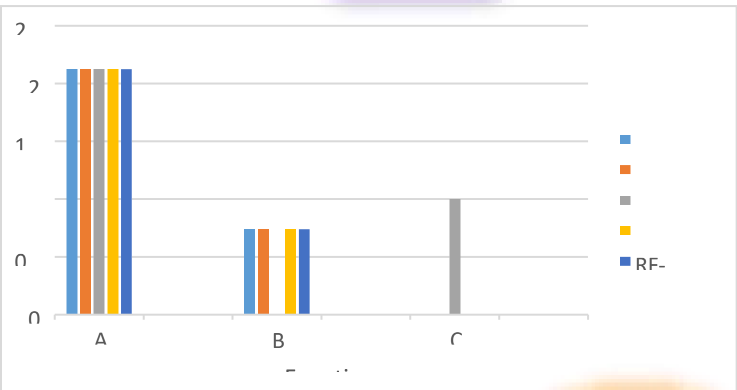


Table 2: Number of iterations

Function	RF-EXP	RF-Halley	Tanh	Arc-sine	RF-arctanh (proposed)
A	3	3	4	2	2
B	3	2	fail	2	2
C	No root	No root	3	No root	No root

Table 3: Approximate roots

Function	RF-EXP	RF-Halley	Tanh	Arc-sine	RF-arctanh
A	2.1253913954	2.1253911988	2.1253912005	2.1253912851	2.1253912857
B	0.7390855632	0.7390851332	fail	0.739051400	0.7390851411
C	No root	No root	1.000000	No root	No root



4. RESULT AND DISCUSSION

As we can see for equation A: arc-sine and RF-arctanh algorithm gives equal iteration but RF-Arctanh give less error,

For equation B: RF-Halley, Arc-sine and RF-Arctanh gives same iterations but RF-Halley give exact root value.

For equation C: only Tanh algorithm gives the root while other methods fails.

- For exponential equation

RF-Arctanh>Arc-sine> RF-Halley>RF-EXP>Tanh

- For trigonometric equations

RF-Halley>RF-Arctanh>Arc-sine>RF-EXP>Tanh

- For logarithmic equations

Tanh>RF-EXP, RF-Halley, RF-Arctanh, Arc-sine

5. CONCLUSION

The Numerical findings and graphical representation of results conclude that the all methods RF-EXP, RF-Halley, Arc-sine, Tanh and RF-ArcTanh are in favour for exponential equation but RF-ArchTanh and Arc-sine gives rapid convergence. Tanh method fails for trigonometric equation whereas it is most suitable for logarithmic equation.

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