Modeling and Performance Analysis of Milk Food Plant Subunits Under Preemptive Priority Repair Policy

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Abstract

This paper presents a reliability model for the availability analysis of a milk food plant composed of three critical subunits. The system is maintained by a single repairman who inspects and repairs units as failures occur. The availability of the milk plant is evaluated using the Regenerative Point Graph Technique (RPGT), considering various failure and repair rates of the subsystems. The study includes graphical representations illustrating the behavior of key performance indicators such as Mean Time to System Failure (MTSF), system availability, and the busy period of the repairman. A real-world milk food plant located in Haryana, serves as the case study for this analysis.

Keywords: Milk Food Plant, Availability Analysis, Regenerative Point Graph Technique, Mean Time to System Failure (MTSF), Reliability Modeling

1. Introduction:

The reliability and availability of industrial process systems have become critical to ensuring optimal production efficiency and reducing maintenance costs. This paper focuses on the transient behavior analysis of a repairable milk food plant using the Regenerative Point Graph Technique (RPGT), an advanced stochastic modeling method grounded in Markov process theory. RPGT facilitates the evaluation of various system parameters such as failure rates, repair rates, system availability, and Mean Time to System Failure (MTSF), enabling a comprehensive understanding of the system's dynamic behavior under different operational conditions. In modern manufacturing and processing industries, particularly in food processing plants like milk food production, availability and maintainability analysis have gained significant importance. These analyses help drive higher productivity levels by minimizing downtime and optimizing maintenance activities. The performance of such plants is often characterized by multiple subunits that operate simultaneously or in standby modes. Understanding the interactions between these subsystems and their impact on overall system reliability is essential for designing effective maintenance policies and ensuring continuous production. This study considers a milk food plant located in Haryana, focusing on three key subunits: Bulk Milk Cooler, Pasteurization Unit, and Pouch Making and Bottle Filling Machine. The plant's subsystem is modeled as a continuous processing and production system, applying a preemptive resume priority repair policy to prioritize repairs and minimize system downtime. The failure and repair rates of the units are assumed constant, and transient probabilities under the Markov process framework are used to construct the system's transient state diagram. The modeling incorporates Laplace transforms to calculate mean sojourn times at various states, which are critical for deriving system performance metrics. The RPGT framework further enables sensitivity analysis by varying failure and repair rates of individual units while holding others constant, thereby assesses their influence on key parameters such as availability, MTSF, and server busy periods. Graphical and tabular results are presented to illustrate these effects, providing insights into the robustness and performance optimization of the milk food plant.

2. Literature Review

Reliability modeling of repairable industrial systems has been extensively studied, with Markov processes emerging as a vital tool for analyzing complex multi-component systems. Rajbala et al. (2019) applied system modeling and analysis techniques to an EAEP manufacturing plant, demonstrating the effectiveness of stochastic methods in industrial contexts. Similarly, Kumar et al. (2017) explored behavior analysis in the urea fertilizer industry, highlighting the importance of reliability in continuous process industries. The Regenerative Point Graph Technique (RPGT) has been adopted in recent years for its efficiency in simplifying the analysis of repairable systems without solving complex state equations (Devi, 2019; Kumar et al., 2019). RPGT's use in cold standby systems and priority-based maintenance policies supports the design of maintenance strategies that balance operational availability with resource constraints. Research by Kumar et al. (2018) extended sensitivity analysis to multi-unit standby systems, revealing how varying failure and repair rates impact system reliability. The integration of heuristic optimization approaches, such as Particle Swarm Optimization (PSO), has further enhanced maintenance scheduling and redundancy allocation, as demonstrated by Kumari et al. (2021). In the food processing sector, studies have analyzed the mathematical modeling and behavior of various plants, including bread manufacturing (Kumar et al., 2018) and paper mill washing units (Kumar et al., 2019). These investigations underscore the necessity of robust reliability models to maintain continuous production and reduce maintenance costs. The present study builds upon this body of work by applying RPGT and Markov modeling to a milk food plant, incorporating priority-based repair policies and performing a detailed sensitivity analysis. The constant failure and repair rate assumption aligns with industry practices, while transient probability calculations provide a nuanced understanding of system behavior. This approach offers practical insights for optimizing maintenance policies and improving the overall reliability and availability of milk food processing plants.

3. Assumptions and Notations

1. It is assumed that a single dedicated repair facility is always available to service any failed unit within the system. This ensures that repairs can commence immediately after a failure is detected without any delay caused by resource unavailability.
2. The failure and repair processes of the subsystems are considered statistically independent. This implies that the occurrence of a failure or repair in one unit does not affect the likelihood or timing of failure or repair in other units.
3. All repairs are assumed to be perfect, meaning that once a subsystem is repaired, it is restored to a condition equivalent to a brand-new unit, with no residual faults or decreased reliability.
4. $h_{i:}$ Constant repair rates
5. $g_{i:}$ Constant failure rates

4. System Description

The milk food plant under study operates through a series of interconnected subsystems, each playing a vital role in the processing and packaging of milk products. The system is composed of three primary units:

- Bulk Milk Cooler (Unit A): This unit is responsible for rapidly cooling raw milk from approximately 30°C down to 4°C. The cooling process is critical for preserving milk quality and extending its shelf life, allowing the cooled milk to be stored safely for up to 96 hours before further processing. Efficient cooling reduces microbial growth and helps maintain the milk's freshness.
- Pasteurization Unit (Unit B): In this unit, milk is subjected to pasteurization, a heat treatment process where milk is heated to at least 72°C (160°F) for a minimum of 17 seconds and then rapidly cooled back to 4°C. This process effectively eliminates harmful pathogens and bacteria, ensuring the milk are safe for human consumption without significantly affecting its nutritional value or taste.

- Pouch Making and Bottle Filling Machine (Unit M): After pasteurization, the milk is transferred to a storage tank linked to automated packaging machinery. The pouch-making machine produces plastic pouches in various sizes to package the milk efficiently. In parallel, the bottle filling machine fills milk into bottles of different volumes, ensuring hygienic, rapid, and consistent packaging suitable for distribution and retail.

5. Transition Diagrams:

Taking into account the above assumptions and system components, the system's behavior is modeled using a transition state diagram based on Markov processes. This diagram, illustrated in Figure 1, captures all possible states of operation and failure among the three subunits, as well as the transitions resulting from failures and repairs over time. The transition diagram provides a visual and mathematical foundation for evaluating system reliability, availability, and performance metrics through the Regenerative Point Graph Technique (RPGT).

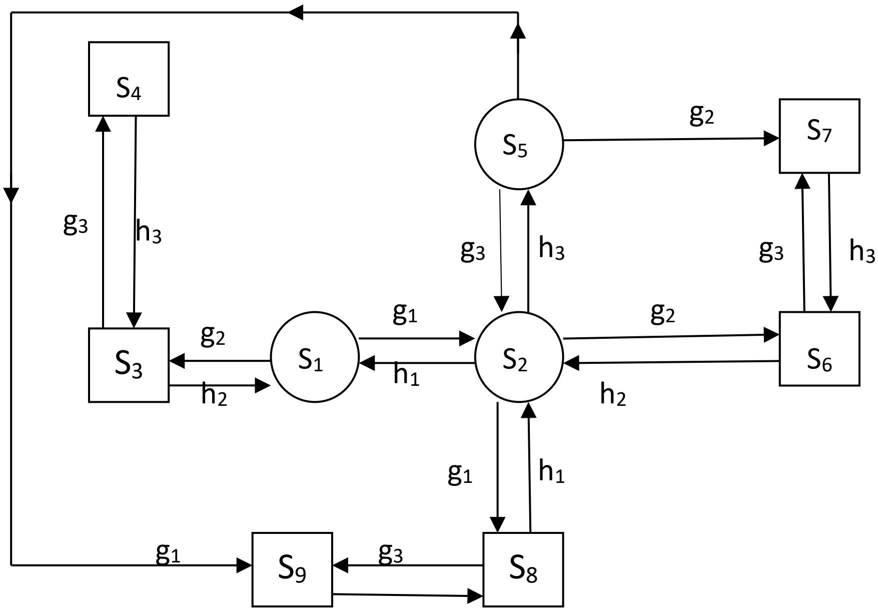

Fig.1: Transition Diagrams

S_1 = A(A)B, S_2 = aAB, S_3 = A(A)b, S_4 = A(A)bM,
S_5 = aABM, S_6 = aAb, S_7 = aAbM, S_8 = aaB,
S_9 = aaBM,

6. Modeling System Parameters

Modeling key system parameters such as Mean Time to System Failure (MTTF), availability, server busy period, and expected number of server visits is essential for assessing the performance and reliability of repairable systems. Using regenerative point graph techniques (RPGT) combined with Markovian state transitions, these parameters can be accurately quantified by analyzing the system's transition probabilities and mean sojourn times in various operational and failure states. For example, MTTF measures the expected operational duration before failure, while availability represents the proportion of time the system remains functional. The server busy period indicates the repairman's workload, and the expected number of server visits reflects maintenance frequency. Together, these metrics provide comprehensive insights into system behavior, enabling effective maintenance planning and optimization of repair resources to enhance overall system reliability and productivity.

Mean time to system failure (T_0): Regenerative un-failed states to which the framework can transit (initial state '2'), earlier incoming any fizzled state are: 'i' = 1, 2, 5 attractive 'ξ' = '1'
$T_0 = (V_{1,1}\,\mu_1 + V_{1,2}\,\mu_2 + V_{1,5}\,\mu_5)/\ \{1 - (1, 2, 1)\}$

Availability of the system (A_0): Regenerative states at which the framework is accessible are 'i' = 1, 2, 5 attractive 'ξ' = '1' whole fraction of time for which the framework is accessible is assumed by

$$A_0 = \left[\textstyle\sum_j V_{\xi,j}, f_j, \mu_j\right] \div \left[\textstyle\sum_i V_{\xi,i}, f_j, \mu_i^1\right]$$

$$A_0 = (V_{2,1}\,\mu_1 + V_{2,2}\,\mu_2 + V_{2,5}\,\mu_5)/Z_1$$

$$\therefore Z = V_{1,1}\,\mu_1 + V_{1,2}\,\mu_2 + V_{1,3}\,\mu_3 + V_{1,4}\,\mu_4 + V_{1,5}\,\mu_5 + V_{1,6}\,\mu_6 + V_{1,7}\,\mu_7 + V_{1,8}\,\mu_8 + V_{1,9}\,\mu_9$$

$$\therefore Z_1 = V_{2,1}\,\mu_1 + V_{2,2}\,\mu_2 + V_{2,3}\,\mu_3 + V_{2,4}\,\mu_4 + V_{2,5}\,\mu_5 + V_{2,6}\,\mu_6 + V_{2,7}\,\mu_7 + V_{2,8}\,\mu_8 + V_{2,9}\,\mu_9$$

Server of busy period (B_0): Regenerative states where server is busy are $2 \leq j \leq 9$, attractive ξ = '1', whole fraction of time for which server remains eventful is by equation

$$B_0 = \left[\textstyle\sum_j V_{\xi,j}, n_j\right] \div \left[\textstyle\sum_i V_{\xi,i}, \mu_i^1\right]$$

$$B_0 = (V_{1,2}\,\mu_2 + V_{1,3}\,\mu_3 + V_{1,4}\,\mu_4 + V_{1,5}\,\mu_5 + V_{1,6}\,\mu_6 + V_{1,7}\,\mu_7 + V_{1,8}\,\mu_8 + V_{1,9}\,\mu_9)/D$$

$$= 1 - (\mu_1/ D)$$

Expected number of server visit's (V_0): Regenerative states where repair man does this job j = 2, 5 taking 'ξ' = '1', number of visit by repair man is given by

$$V_0 = \left[\textstyle\sum_j V_{\xi,j}\right] \div \left[\textstyle\sum_i V_{\xi,i}, \mu_i^1\right]$$

$$V_0 = (V_{1,2} + V_{1,5})/ D$$

7. Results and Discussions:

Behavior Analysis: Particular Cases: - $h_i = h$;$g_i = g$

Mean time to system failure [MTSF] (T_0): The analysis of key performance metrics, including Mean Time to System Failure (MTTF), system availability, server busy period, and the expected number of server visits, provides valuable insights into the behavior of the repairable milk food plant under varying failure and repair rates. Figure 2 illustrate the MTTF values for different repair rates (h) and failure rates (g). As observed, the MTTF decreases as the failure rate increases from 0.15 to 0.35, indicating a shorter expected operational time before system failure. Conversely, increasing the repair rate from 0.55 to 0.75 results in a slight decrease in MTTF, suggesting that while repair efficiency improves, it must be balanced with failure occurrences. This trend aligns with practical expectations, where higher failure rates reduce system reliability, and improved repairs can only partially offset this effect.

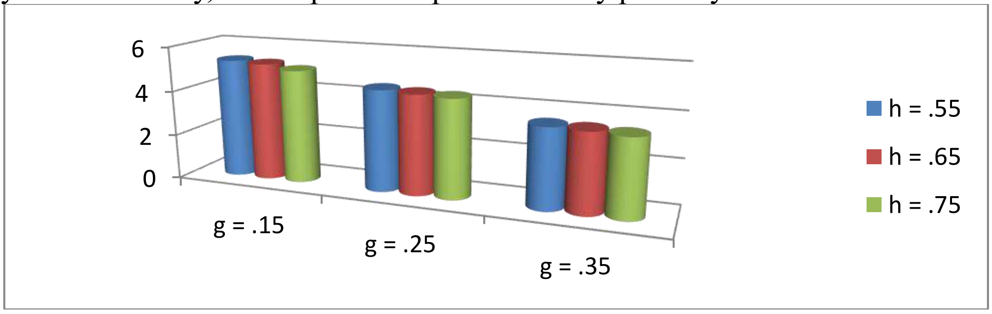

Figure 2: MTSF

From the above figure 2 one can determine that MTSF is increasing which must be so once the repair rate amassed and decreases when the disappointment rate raises which should be so in practical situations.

Availability of the system (A_0): Figure 3 depicts the system availability under the same parameter variations. Availability increases significantly as the repair rate improves, rising from 0.84 to 0.93 when the failure rate is low (0.15). However, availability declines with higher failure rates, dropping to 0.62 when the failure rate reaches 0.35, even at higher repair rates. This indicates that maintaining low failure rates is crucial to ensure consistent system uptime. The availability results reinforce the practical need for both effective maintenance and robust system design to minimize failures.

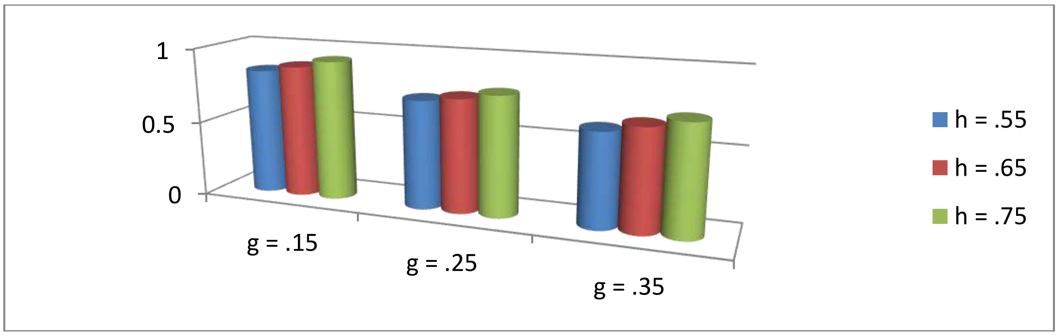

Figure 3: Availability of the system

The above Figure 3 shows that the Availability is increasing when the repair rate is increasing and decrease with the rise in disappointment rate, which ought to be actually.

Server of the busy period (B_0): The server busy period shown in Figure 4, reflects the fraction of time the repairman is actively engaged. The busy period decreases as repair rates increase, implying that efficient repairs reduce the load on maintenance resources. However, as failure rates rise, the busy period increases, highlighting the additional strain placed on the repairman during frequent breakdowns. This information is vital for resource planning, ensuring that sufficient repair capacity is available during high failure conditions to prevent bottlenecks.

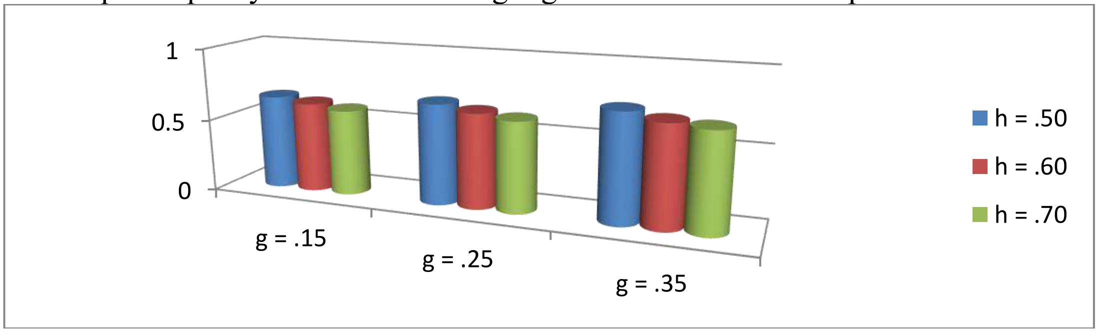

Figure 4: Server of the busy period

It can be concluded from the above figure 4 that the values of server of busy period shows the expected trend for various values of disappointment rate, as server of busy period decreases with the rise in the values of repair rate.

Expected number of server visits (V_0): Figure 5 present the expected number of server visits, which rises with increasing repair rates and failure rates. A higher repair rate naturally leads to more frequent maintenance completions, while higher failure rates cause more breakdowns requiring repair visits. Understanding this relationship helps in forecasting maintenance workload and planning repair schedules to optimize system availability and cost-effectiveness.

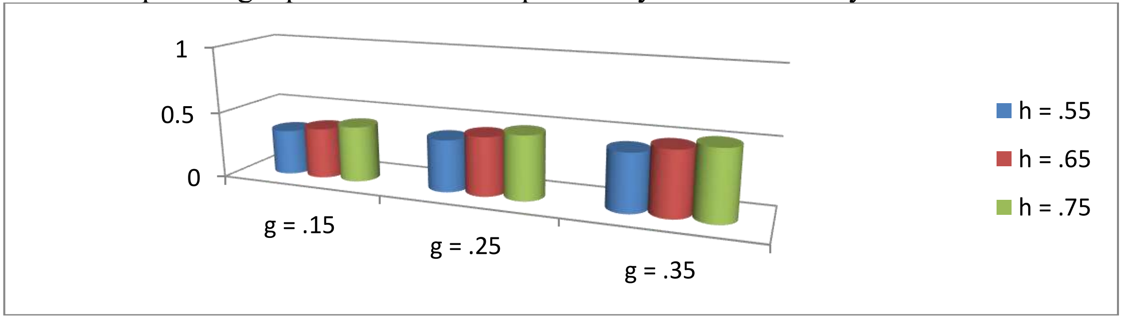

Fig. 5: Expected number of server visits

It can be concluded from the above figure 5 and table 4that the values of Expected number of server visits demonstrations the expected trend for various values of disappointment rate, as Expected number of server visits increases with the rise in the values of repair rate.

8. Conclusion

In conclusion, the comprehensive analysis of these system parameters demonstrates the interdependent effects of failure and repair rates on system reliability and maintenance demands. To achieve optimal system performance, management must carefully control failure rates through robust design and preventive measures, while simultaneously enhancing repair capabilities to reduce downtime. Financial resources and market conditions will influence the feasible balance, but strategic optimization of these parameters is essential for sustaining high productivity and minimizing operational costs in the milk food plant.

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