Reliability and Availability Assessment of a Biscuit Production System Through Markov Modeling

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Abstract

Standby systems are widely utilized across various industries to enhance system reliability and availability. Parallel redundancy occurs when multiple backup components operate simultaneously, ensuring continuous operation despite failures. These standby units are generally classified as hot, warm, or cold, depending on their load conditions relative to the primary unit. Hot standbys operate under similar loads as active units, offering immediate backup. This study applies the Regenerative Point Graph Technique (RPGT) to perform a sensitivity analysis on a biscuit manufacturing plant composed of three key units with constant failure and repair rates. The analysis investigates how variations in failure and repair rates of individual units influence overall system performance metrics such as Mean Time to System Failure (MTSF) and availability. The results are presented through detailed tables and graphs, providing insights into the system's reliability behavior under different operational scenarios.

Keywords: Mean Time to System Failure (MTSF), Regenerative Point Graph Technique (RPGT), Sensitivity Analysis, System Availability, Markov Modeling

1. Introduction:

Standby systems play a crucial role in enhancing the reliability and availability of industrial processes by providing backup units that can take over operations in case of primary unit failure. These systems are common in manufacturing environments where continuous operation is critical, such as biscuit production plants. Redundancy through standby units is typically categorized as hot, warm, or cold, depending on whether the standby units share the load with active units or remain idle until needed. Hot standby units operate with the same load as active units, enabling immediate backup without delay. This study focuses on the reliability and availability assessment of a biscuit production system using Markov modeling and the Regenerative Point Graph Technique (RPGT). The system under consideration consists of three primary units, each with constant failure and repair rates. Through sensitivity analysis, this work investigates how variations in failure and repair rates affect key performance indicators like Mean Time to System Failure (MTSF) and system availability. The findings provide insights that can guide maintenance policies and system design to optimize operational efficiency in biscuit manufacturing plants.

2. Review of Literature

Reliability and availability modeling of standby systems has been extensively studied due to its importance in ensuring continuous industrial operations. Markov processes have been widely used to model reliability in repairable systems, enabling the derivation of performance metrics such as availability and MTSF (Hua & Ping, 2018). The Regenerative Point Graph Technique (RPGT), an extension of Markov modeling, offers an efficient approach to analyze complex systems without the need for solving extensive state equations (Devi, 2019; Gupta & Devi, 2020). RPGT has been applied successfully to various manufacturing systems, including those with multi-unit configurations and standby redundancies (Garg & Garg, 2022). Sensitivity analysis plays a vital role in understanding the impact of system parameters such as failure and repair rates on overall system reliability and availability (Ahmad, 2015; Agrawal et al., 2021). These analyses help in identifying critical components and optimizing maintenance schedules to enhance system uptime and reduce costs. Recent studies emphasize the need for adaptive maintenance strategies in modern manufacturing environments to address uncertainties and dynamic operational conditions (Singh & Gahlot, 2021; Torrado & Jorge, 2021). Overall, the literature indicates that combining Markov modeling with RPGT and sensitivity analysis provides a robust framework for evaluating and optimizing the reliability of repairable systems in industrial applications, including biscuit manufacturing plants.

3. Notations &Assumptions

- Two of the system's four units are currently in operation, while the other two are in cold standby mode.
- The system consists of four units, with two units actively operating and the remaining two units held in cold standby mode.
- The system is considered to have failed if more than two units malfunction simultaneously.
- Each unit (i) (where (i = 1, 2, 3, 4)) has a constant repair rate denoted by (w_i).
- Each unit (i) has a constant failure rate denoted by (λ_i).
- The system operates at full capacity when both active units are functioning properly.
- The system operates at reduced capacity when one or more units are in standby or undergoing repair.
- Failed states represent system conditions where units are non-operational.
- Regenerative points are specific states in the system where the process probabilistically restarts, used for analysis in regenerative process theory.

4. System Transition State Diagram

Figure 1 illustrates the system's transition state diagram modeled using the Markov process. This diagram reflects the states and transitions of the system based on the assumptions and notations outlined above. Each state represents a configuration of operational and failed units, while transitions correspond to failures and repairs occurring over time.

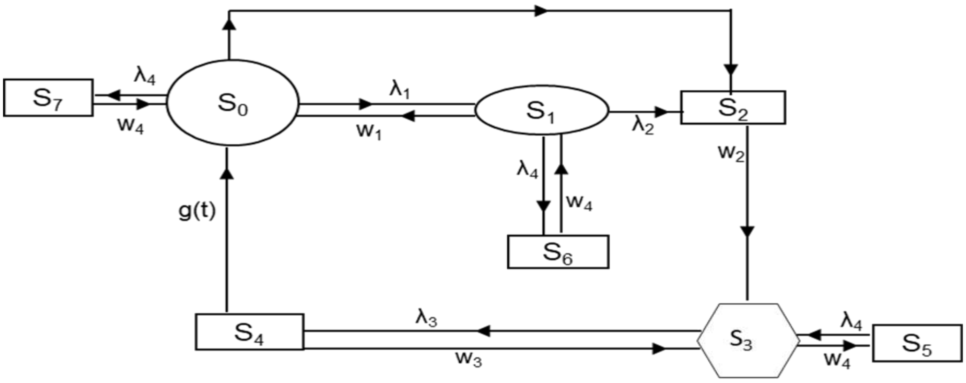

Fig. 1: Transition Diagram of Biscuits Plant

$S_0 = AB$ $S_1 = \bar{A}B$ $S_2 = aB$
$S_3 = \bar{A}_1B$ $S_4 = a_1B$ $S_5 = \bar{A}_1b$
$S_6 = \bar{A}b$ $S_7 = Ab$

5. Data analysis and Discussions

MEAN TIME TO SYSTEM FAILURE (MTSF) (T_0): The regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: 'i' = 0,1,2,3,4,5,6,7 taking 'ξ' = '0'.

$$\text{MTSF} = \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi\xrightarrow{s_r(sff)}i\right)\right\}.\mu_i}{\prod_{k_1\neq\xi}\left\{1-V_{\overline{k_1,k_1}}\right\}}\right\}\right] \Big/ \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(\xi\xrightarrow{s_r(sff)}\xi\right)\right\}}{\prod_{k_2\neq\xi}\left\{1-V_{\overline{k_2,k_2}}\right\}}\right\}\right]$$

AVAILABILITY OF THE SYSTEM: The regenerative states at which the system is available are 'j' = *0,1,3*and the regenerative states are '*i*' = *0* to *4* taking 'ξ' = '*0*' for which the system is obtainable is given by

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow}j)\}fj,\mu j}{\Pi_{m_{1\neq\xi}}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] / \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow}i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$A_0 = \left[\sum_j V_{\xi,j}\,, f_j, \mu_j\right] / \left[\sum_i V_{\xi,i}\,, f_j, \mu_i^1\right]$$

BUSY PERIOD OF THE SERVER: The recreating states where attendant ‘j’ = 1,2,3,4 & ξ = ‘0’, on behalf of which the attendant remains hectic is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow}j)\},nj}{\Pi_{m_{1\neq\xi}}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] / \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow}i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$B_0 = \left[\sum_j V_{\xi,j}\,, n_j\right] / \left[\sum_i V_{\xi,i}\,, \mu_i^1\right]$$

EXPECTED FRACTIONAL NUMBER OF REPAIRMAN’S VISITS (V_0): The renewing states somewhere the reparation man ensure this job$j = 1$&‘ξ’ = ‘0’, the integer of stopover by the restoration man is specified by

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow}j)\}}{\Pi_{k_{1\neq\xi}}\left\{1\text{-}V_{\overline{k_1 k_1}}\right\}}\right\}\right] / \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow}i)\}\mu_i^1}{\Pi_{k_{2\neq\xi}}\left\{1\text{-}V_{\overline{k_2 k_2}}\right\}}\right\}\right]$$

$$V_0 = \left[\sum_j V_{\xi,j}\right] / \left[\sum_i V_{\xi,i}\,, \mu_i^1\right]$$

PROFIT FUNCTION: Profit Function can be done by utilizing the optimization function

$P_0 = D_1A_0 - D_2B_0 - D_3V_0$

Where: $D_1 = 1000$; $D_2 = 100$; $D_3 = 50$

The profit function has been analyzed across a range of failure and repair rates to understand its sensitivity to these parameters. Specifically, failure rates ((\lambda)) were varied through values of 0.05, 0.10, 0.15, 0.20, 0.25, and 0.30, while repair rates ((w)) were adjusted across 0.83, 0.86, 0.89, 0.92, 0.95, and 0.98. The results obtained from this analysis are summarized in Table 5.10 and illustrated graphically in Figure 2. The data reveal a distinct trend: as failure rates increase from 0.05 to 0.30, the profit function value for the system correspondingly rises. This suggests that higher failure rates, which typically degrade system performance, are associated with increased costs or losses reflected in the profit function. Conversely, when repair rates improve, increasing from 0.83 to 0.98, the profit function value decreases, indicating enhanced system profitability due to more efficient repairs reducing downtime and maintenance costs.

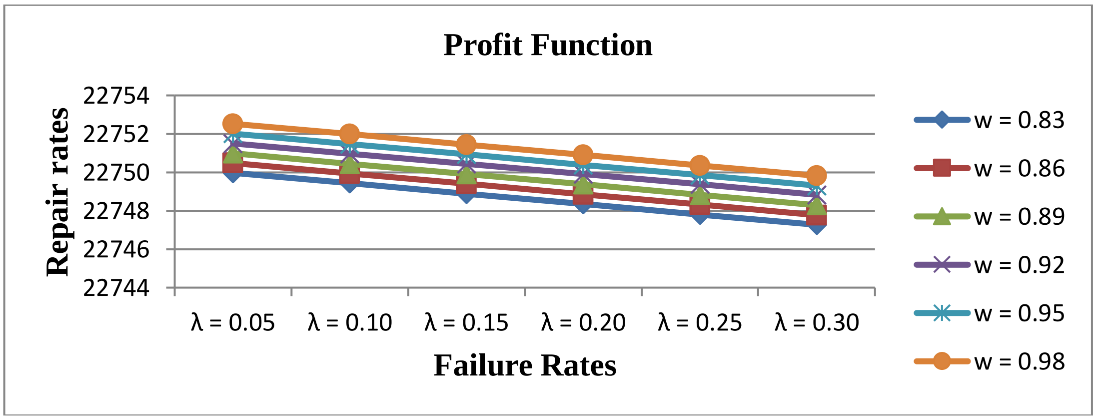

Figure 2: Profit function

The ideal or optimal value of the profit function, quantified as 22,752.52, is achieved under the condition of minimizing the failure rate to its lowest tested value (0.05) while maximizing the repair rate to the highest value considered (0.98). This optimal point underscores the critical balance between reducing failures and improving repair efficiency to maximize the economic performance of the system. In summary, the analysis highlights that system profitability is highly sensitive to failure and repair rates, emphasizing the importance of maintaining low failure occurrences and high repair effectiveness to achieve optimal operational and financial outcomes.

6. Conclusion

This study successfully employed Markov modeling and the Regenerative Point Graph Technique (RPGT) to assess the reliability and availability of a biscuit production system consisting of multiple standby units. Through sensitivity analysis, the impact of varying failure and repair rates on system performance metrics such as Mean Time to System Failure (MTSF), availability, and server busy periods was thoroughly investigated.

A key contribution of this work is the analysis of the profit function in relation to these parameters. The results indicate that system profitability is highly sensitive to failure and repair rates. Specifically, increasing failure rates lead to higher costs and reduced profit, while improving repair rates enhances system profitability by minimizing downtime and maintenance expenses. The optimal profit value was achieved by minimizing failure rates and maximizing repair efficiency, highlighting the importance of robust system design and effective maintenance strategies. Overall, the findings provide valuable insights for decision-makers in the biscuit manufacturing industry to optimize system reliability and availability while controlling operational costs. The integration of profit function analysis with reliability modeling offers a comprehensive framework for balancing technical performance and economic objectives, ultimately supporting more informed and cost-effective maintenance planning.

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