A Simplified Approach to Optimal Piezoelectric Patch Placement Using Genetic Algorithm for Active Vibration Control of Plates

Yusuf Khan, Research Scholar, Department of Mechanical Engineering, Anjuman-I-Islam's Kalsekar Technical Campus, New Panvel, Navi Mumbai- 410206, Maharashtra, India., yousufkhan86@gmail.com yusuf.khan@aiktc.ac.in
S. M. Khot, Department of Mechanical Engineering, "Agnel Charities" Fr. C. Rodrigues Institute of Technology, Vashi, Navi Mumbai, 400 703, Maharashtra, India., smkhot66@yahoo.co.in, smkhot@fcrit.ac.in
Vishal G. Salunkhe, Department of Mechanical Engineering, "Agnel Charities" Fr. C. Rodrigues Institute of Technology, Vashi, Navi Mumbai, 400 703, Maharashtra, India., vishalsalunkhe84@gmail.com, vishal.salunkhe@fcrit.ac.in

Purpose

The purpose of this study is to present a simplified methodology for determining the optimal placement of piezoelectric patches on flexible plate-type structures, aimed at enhancing active vibration control efficiency.

Methods

The methodology utilizes the displacement eigenfunction to identify points of maximum strain, minimizing computational complexity. A Genetic Algorithm (GA) is applied to optimize patch placement by using strain distribution as the objective function. The study focuses on a pinned support rectangular plate, analyzing its strain profile to identify optimal locations for multiple piezoelectric patches. These identified locations are further validated using controllability and observability considerations.

Results

The results demonstrate that the proposed methodology effectively pinpoints optimal patch positions based on the plate's specific x and y coordinates, significantly improving vibration control performance across multiple modes.

Conclusion

The research provides a practical and efficient approach for optimizing piezoelectric patch placement, contributing to active vibration control technology advancements for flexible plate-type structures.

Abstract

Effective active vibration control of flexible plate-type structures relies heavily on the optimal placement of piezoelectric actuators and sensors. This paper presents a simplified and control-independent methodology for determining optimal patch locations to maximize control efficiency. The approach employs displacement eigenfunctions to identify regions of maximum strain, which serve as the objective function in a Genetic Algorithm (GA)-based optimization framework. A rectangular plate with pinned supports is considered as a case study, where multimode strain profiles are analyzed to establish the most effective patch locations. To ensure reliability, the GA-derived results are validated against controllability and observability indices. The findings demonstrate that identifying specific x–y coordinates of patch centers significantly improves vibration suppression performance. The proposed methodology reduces computational complexity, ensures accurate placement of actuators and sensors, and offers a practical framework for advancing active vibration control in flexible plate-type structures.

Keywords: Active vibration control, optimal placement, flexible structure, genetic algorithm, optimal control.

1. Introduction

Recent advancements in space structures, aircraft, and robotic manipulators have driven the need for lightweight, flexible designs with low modal frequencies and damping ratios. However, these structures often suffer from higher flexibility, reduced damping, and prolonged vibration suppression times compared to rigid structures [1]. The findings highlight the importance of actively controlling vibration in large space structures, where precise control is critical. Smart structures featuring optimized actuator and sensor placements are essential for addressing these challenges. Researchers typically divide control methods into active and passive categories based on the disturbance frequency. Active methods address disturbances below 1000 Hz, while passive methods manage those above 1000 Hz. Active control is often preferred for its effectiveness across varying conditions. Traditionally, passive isolators and dampers, such as rubber mountings and passive dampers, were used to manage mechanical vibrations [2]. Active vibration control has regained popularity due to developments in digital signal processing (DSP) and sensors and actuators. A typical smart structure incorporates sensors to track dynamic behavior, a processor to analyze the data, actuators to carry out the control actions, and a power source to drive the system. The field has gained significant interest due to advancements in space exploration, rapid processors, responsive operating systems, and high-performance sensors and actuators [3, 4]. Optimal active vibration control involves analyzing vibrational properties, identifying ideal sensor and actuator locations, and designing an efficient control system. Researchers have extensively studied placement techniques using the linear quadratic regulator (LQR) to control composite shells with bonded piezoelectric patches [5].

Piezoelectric materials, valued for their rapid response, flexibility, lightweight, and low power use, are key in active vibration control (AVC). These materials convert electrical signals to mechanical strains and vice versa, making them ideal for sensing and actuation. Piezoelectric materials are used as layers, bonded patches, cylindrical stacks, screen-printed layers, fiber composites, and graded material patches. Piezoelectric patches that are implanted or surface-mounted give precise control over structural reactions [6–11]. Effective structural vibration control requires careful consideration of the number and positioning of piezoelectric sensors and actuators. Significant research has focused on designing and placing these components to enhance the control of flexible structures, ensuring maximum performance in vibration suppression [12]. Misplacement of sensors and actuators can result in issues such as reduced observability and controllability and unwanted spillover effects [13]. The efficiency of energy harvesting and vibration damping is highly dependent on the strategic placement of transducers within a structure. Consequently, many studies emphasize optimizing this placement to improve performance. Determining the optimal location for piezoelectric patches in smart structures is crucial for the effective operation of active control systems, attracting interest across research disciplines.

To achieve effective control, strategically place actuators at locations with higher strain in both the time and frequency domains. Optimal active vibration control involves calculating vibrations, placing sensors and actuators, and designing the control system. Many researchers have tackled the challenge of determining optimal sensor and actuator placement. The linear quadratic regulator (LQR) method was utilized for active vibration control of laminated composite shells integrated with bonded piezoelectric patches. Other studies have optimized performance indices using H2 norms [14] and H∞ norms for sensor and actuator positions. Linear quadratic Gaussian (LQG) schemes have optimized sensor and actuator placement, while controllability and observability grammian methods have been used for optimal placement [13].

Han and Lee [15] examined sensor and actuator placement for controllability, observability, and spillover prevention using controllability grammian indices. Dhuri and Seshu [16] focused on maximizing controllability and observability measures for active piezoelectric damping. Kumar and Narayanan [17] explored the use of linear quadratic regulator (LQR) controllers to optimize the placement of piezoelectric patches on beam structures. Fein and Gaul [18] applied analytical strain energy methods to improve the positioning of piezoelectric sensors on 2D plates. Halim and Moheimani [19] proposed an optimization method based on modal and spatial controllability measures for thin plates. Similarly, Lin et al. [20] employed the maximal modal force rule to identify the optimal placement of actuators. Main et al. [21] investigated the effects of piezoelectric patch sizes and locations based on stiffness ratios between piezoceramic materials and substrate structures. Gupta et al. [22] reviewed optimal placement strategies, considering criteria such as modal forces, deflection, control effort, controllability, observability, and spillover minimization. Controllability, observability, energy dissipation, and system stability are key optimization objectives in placing piezoelectric sensors and actuators.

Several studies have emphasized the application of genetic algorithms to determine optimal configurations for piezoelectric systems [15, 23-24]. These algorithms excel at handling non-convex search spaces involving both continuous and discrete optimization variables within multi-objective frameworks. Mehrabian and Yousefi-Koma [25] utilized bio-inspired approaches, incorporating finite element analysis and neural networks, to determine optimal configurations for piezo actuators. Xu and Jiang [26] utilized genetic algorithms to optimize the placement of piezoelectric elements within truss structures.

Plates play a vital role as structural elements in modern construction, as well as in aerospace and aeronautical engineering. Various models for plate analysis based on different theories are investigated in the literature. Kirchhoff plate theory is typically employed for the dynamic modeling thin plates, while Mindlin plate theory is utilized for thick plates. Gao et al. [27] studied the vibration and acoustic behavior of a supported thin plate, focusing on harmonic point forces or incident plane waves as primary noise sources. They also explored the use of piezoelectric patches for sound level control. Caruso et al. [28] investigated vibration control in an elastic plate clamped along one edge, focusing on its response to an impulsive transverse force applied at a free corner. Lam et al. [29] proposed a finite element model incorporating piezoelectric sensors and actuators for active vibration control, further validated using a cantilever composite plate. Narayanan and Balamurugan [30] developed a finite element approach for modeling laminated structures with integrated piezoelectric layers, accounting for the effects of stiffness, mass, and electromechanical coupling in the piezoelectric laminates. Tzou and Fu [31] developed dynamic models to analyze the vibration response of supported elastic rectangular plates with variable rectangular piezoelectric patches. In flexible structures, mapping physical displacements onto a modal basis enables independent monitoring and control of each mode through individual sensors and actuators. Daraji et al.[32] identified optimal patch positions on cantilever plate structures by measuring sensor effectiveness as a percentage. The methodology involved normalizing each sensor's output voltage by dividing it by the maximum output for each mode, utilizing both time and frequency domain analyses. This approach was applied to dynamically symmetric and asymmetric structures under external forces and base excitations. The approach optimized sensor and actuator distribution using time and frequency response analysis, comparing results with published findings for cantilever plates.

The provided literature review highlights the challenges associated with existing methodologies for optimizing the placement of piezoelectric patches on structures for vibration control. These approaches are time-consuming and computationally complex due to the tedious math and high cost of handling many elements. Mode shapes are inherent to structures, with displacement patterns remaining consistent for each mode, while the magnitude varies with different modal displacements. The strain profile follows the displacement eigenfunction of the mode shape, and regions of maximum strain are the most effective areas for piezoelectric patch placement. The displacement eigenfunction of the structure’s mode shapes can be differentiated to determine the regions of maximum strain. Since these strain regions do not change for a given mode shape, they provide an efficient means of identifying the optimal locations for piezoelectric patches.

The proposed hypothesis suggests a simplified approach for identifying optimal piezoelectric patch locations by leveraging the displacement eigenfunction for a plate-type structure. This methodology assumes that maximum strain regions, which are crucial for effective vibration control, can be determined by analyzing the displacement eigenfunction. The steps in this approach would include:

Utilizing Displacement Eigenfunction: The displacement eigenfunction of a mode shape is a reliable indicator of strain patterns. By differentiating the eigenfunction, regions of maximum strain can be pinpointed.

1. Objective Function Based on Kirchhoff Plate Theory: For thin plates with small deflections, Kirchhoff plate theory will derive an objective function that accurately describes the strain distribution across the plate.

2. Simplification of Mathematical Formulations: The new methodology reduces mathematical complexity by focusing on strain regions derived from displacement eigenfunctions.

3. Efficiency in Computation: This methodology avoids full-scale optimization of actuator positions, making it more computationally efficient. It is particularly suited for systems with many degrees of freedom**.**

The study presents an efficient solution for optimal piezoelectric patch placement using strain distribution from mode shapes and displacement eigenfunctions. **This method simplifies the optimization process, making it well-suited for complex structures with numerous degrees of freedom. The proposed approach is validated by comparing its accuracy and computational efficiency with other existing methods, highlighting its effectiveness for real-world applications.**

2. Methodology

The placement of actuators and sensors plays a crucial role in effectively controlling vibrations in plate-type structures. Piezoelectric patches (piezopatches) are widely used as actuators and sensors in aerospace, automotive, and civil engineering for their compact actuation and sensing capabilities. Determining the optimal placement of piezopatches is complex and computationally intensive, particularly for large structures with dynamic behavior.

This study aims to develop a simplified methodology for the optimal placement of piezo patches in plate-type structures. The proposed approach focuses on identifying the ideal locations by targeting points of maximum strain, which is critical for effective vibration control. The methodology uses displacement eigenfunctions to reduce computational time and complexity while ensuring optimal piezopatch placement for maximum effectiveness. The streamlined process proposed in this study offers a practical solution for engineers and researchers working on vibration control in plate-type structures. The proposed methodology improves the efficiency of the design process while ensuring that the placement of actuators and sensors achieves optimal performance in vibration suppression. The flowchart in Figure 1 illustrates the algorithm used to accomplish this.

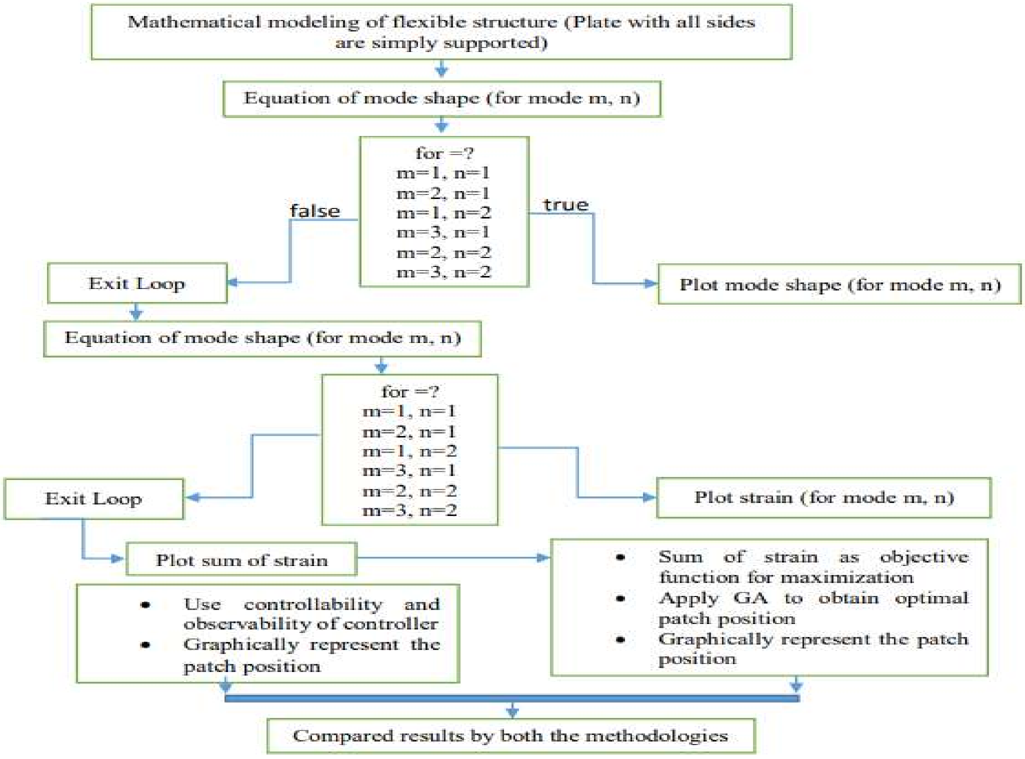


Fig. 1 Flow chart of the proposed methodology

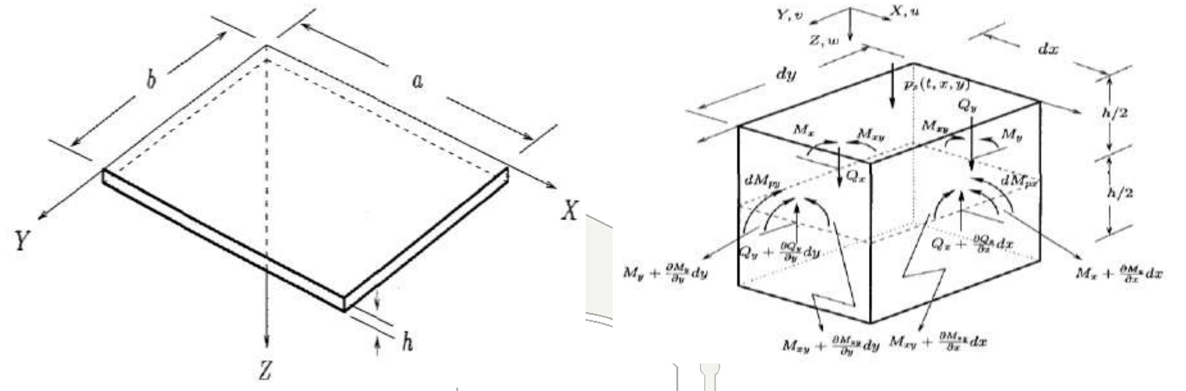
2.1 Modeling of the Flexible Plate Structure:

The section focuses on developing dynamic models for a flexible plate structure where all sides are pinned and supported. The Kirchhoff plate theory derives the dynamic model, which applies to thin plates with small deflections. The assumption of small deflections allows the use of a small angle approximation, significantly simplifying the derivation process and enabling the linearization of the equations of motion. Several key assumptions are made in the modeling process to ensure the validity of the derived equations [33].

1.The plate is of uniform thickness.

2. Shear deformation, lateral stress, and rotational inertia are neglected.

The plate under study is a thin structure with dimensions a × b × h, where its behavior is defined by material properties such as Young's modulus (E), density (ρ), and Poisson's ratio (ν). The coordinate system is aligned with the plate's mid-plane, referred to as the neutral surface, where longitudinal bending strain is absent. A diagram of this plate element is shown in Figure 2.



(a) (b)

Fig. 2 (a) Thin plate in transverse vibration, (b) small element of the plate [33]

The deflection along the X and Y axes can be described

$u = -z\frac{\partial w}{\partial x}$,	(1)
$v = -z\frac{\partial w}{\partial y}$.	

Where w is deflection in z direction. The strains can be derived from Eq. (1) using Hooke's Law.

$\epsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2}$,	(2)
$\epsilon_y = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2}$, and	
$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w}{\partial x \partial y}$.	

Where γ_{xy} represents the shear strain, while ϵ_x and ϵ_y denote the longitudinal strains in X and Y directions respectively. According Hooke's Law, these strains are related to stresses as follows:

$\epsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y)$,	(3)
$\epsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x)$, and	
$\gamma_{xy} = \frac{1}{G}\tau_{xy} = 2\frac{(1+v)}{E}\tau_{xy}$.	

The shear modulus G is related to Young's modulus as expressed above. The shear stress is represented by τ_{xy} while γ_x and γ_y correspond to longitudinal stresses in the X and Y directions respectively. Consequently, the stresses can be derived from Eq. (2) and (3).

$\sigma_x = -\frac{Ez}{(1-v^2)}\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right)$,	(4)

$$\sigma_y = -\frac{E\,z}{(1-v^2)}\left(\frac{\partial^2 w}{\partial y^2} + v\,\frac{\partial^2 w}{\partial x^2}\right), \text{and}$$

$$\tau_{xy} = -\frac{E\,z}{(1-v)}\,\frac{\partial^2 w}{\partial x\,\partial y}.$$

The moment per unit length, M_x , is determined by integrating the allied stress across the thickness of the plate,

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\,\sigma_x\,dz = -D\left(\frac{\partial^2 w}{\partial x^2} + v\,\frac{\partial^2 w}{\partial y^2}\right) \tag{5}$$

In this case, D denotes the flexural rigidity of the plate.

$$D = \frac{E\,h^3}{12\,(1-v^2)}. \tag{6}$$

Similarly, for M_y,

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\,\sigma_y\,dz = -D\left(\frac{\partial^2 w}{\partial y^2} + v\,\frac{\partial^2 w}{\partial x^2}\right) \tag{7}$$

The torsional moment per unit length M_{xy},

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\,\tau_{xy}\,dz = -D\,(1-v)\,\frac{\partial^2 w}{\partial x\,\partial y} \tag{8}$$

Where dM_{px} and dM_{py}, represent the external moments per unit length, and p_z represents the pressure in the Z direction. As illustrated in Fig. 2, Q_x and Q_y indicate for the shearing forces per unit length. By considering the vertical forces in the Z direction and applying Newton's second law,

$$Q_x\,dy - \left(Q_x + \frac{\partial Q_x}{\partial x}\,dx\right)dy + Q_y\,dx - \left(Q_y + \frac{\partial Q_y}{\partial y}\,dy\right)dx + p_z\,(t,x,y)\,dx\,dy - \rho\,(x,y)\,h\,dx\,dy\,\frac{\partial^2 w}{\partial t^2} = 0. \tag{9}$$

Dividing by dx dy yields:

$$-\frac{\partial Q_x}{\partial x} - \frac{\partial Q_y}{\partial y} + p_z\,(t,x,y) - \rho\,(x,y)\,h\,\frac{\partial^2 w}{\partial t^2} = 0. \tag{10}$$

Taking the moment equilibrium about the X-axis and neglecting the plate's rotational inertia,

$$p_z\,(t,x,y)\,dx\,dy\,\frac{dy}{2} - \frac{\partial Q_x}{\partial x}\,dx\,dy\,\frac{dy}{2} - \left(Q_y + \frac{\partial Q_y}{\partial y}\,dy\right)dx\,dy - \frac{\partial M_y}{\partial y}\,dy\,dx - \frac{\partial M_{xy}}{\partial x}\,dx\,dy - d\,M_{py}\,dx = 0. \tag{11}$$

Dividing by dx dy and ignoring the higher order term dy,

$$-Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{py}}{\partial y}. \tag{12}$$

In the same way, the moment equilibrium about the Y axis can be found be obtain,

$$-Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} + \frac{\partial M_{px}}{\partial x}. \tag{13}$$

Similarly, it is assumed that Mxy = Myx due to complementary shear stress condition $\tau_{xy} = \tau_{yx}$. Differentiating Eq. (12) with respect to y,

$$-\frac{\partial Q_y}{\partial y} = \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{py}}{\partial y^2}. \tag{14}$$

Differentiating Eq. (12) with respect to x,

$$-\frac{\partial Q_x}{\partial x} = \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{px}}{\partial x^2}. \tag{15}$$

Substituting Eq. (14), (15) in vertical equilibrium Eq. (10).

$$\left(\frac{\partial^2 M_x}{\partial x^2} + 2\,\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_y}{\partial y^2}\right) + \left(\frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2}\right) + p_z\,(t,x,y) - \rho\,(x,y)\,h\,\frac{\partial^2 w}{\partial t^2} = 0. \quad (16)$$

Now, the moment expressions in Equation (5), (7), and (8) can be differentiated twice to obtain $\partial^2 M_x/\partial x^2$, $\partial^2 M_y/\partial y^2$ and $\partial^2 M_{xy}/\partial x\,\partial y$. The partial differential equation of a thin plate under transverse vibration is obtained by simplifying and substituting these into Eq. (16).

$$\rho\,(x,y)\,h\,\frac{\partial^2 w}{\partial t^2} + D\,\nabla^4\,w(t,x,y) = \frac{\partial^2 M_{px}}{\partial x^2} + \frac{\partial^2 M_{py}}{\partial y^2} + p_z\,(t,x,y). \quad (17)$$

Where,

$$\nabla^4\,w = \frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4}. \quad (18)$$

In analyzing a simply supported rectangular plate, the natural frequencies and corresponding mode shapes are crucial for understanding the vibrational characteristics. Each mode shape represents a specific vibration pattern, and it depends on both spatial variables x and y. For a plate with simply supported edges, the mode shapes are defined by the spatial coordinates, with the mode numbers m and n representing the X and Y directions, respectively.

The normalized mode shape for a given mode (m, n) given as,

$$W_{mn}(x,y) = \frac{2}{\sqrt{a\,b\,\rho\,h}}\,\sin\frac{m\,\pi\,x}{a}\,\sin\frac{n\,\pi\,y}{b}, \quad (19)$$

Where the natural frequency of mode (m, n) is:

$$\omega_{m\,n} = \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\sqrt{\frac{D}{\rho\,h}}. \quad (20)$$

2.2 Mode Shape

For a rectangular plate with all sides simply supported, the mode shape for vibration is provided by Equation (19). These mode shapes, defined by the product of sine functions representing the plate's deformation pattern, can be visualized using MATLAB©. Solving the governing differential equation (Eq. 20) for the plate's motion yields the natural frequencies for each mode shape. This solution considers the plate's boundary conditions and physical properties, including material density, elasticity, and geometry. Figure 3, generated using MATLAB©, illustrates the mode shapes for a simply supported plate. These visualizations reveal how the plate deforms and vibrates under different modes, essential for designing effective vibration control strategies.

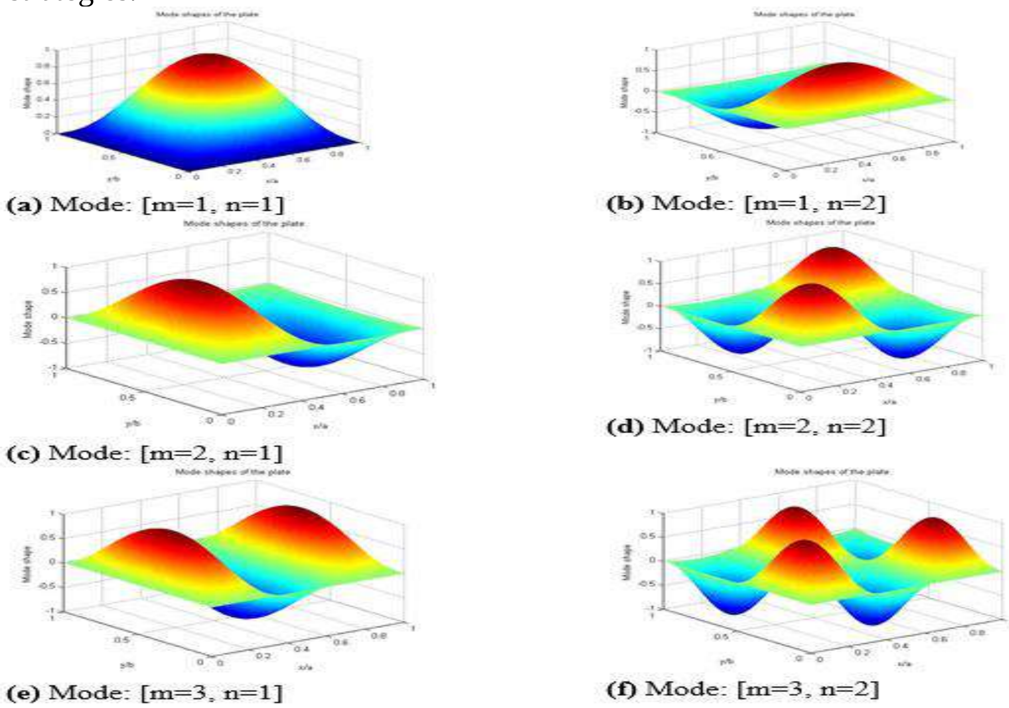


Fig. 3 Mode shapes for a simply supported plate

In the mode shapes, the product of sine functions indicates the number of half-waves in each direction (X and Y). For example, the mode shape corresponding to m=1 and n=1 would have one half-wave in each direction, representing the simplest vibration mode. As the values of m and n increase, the mode shapes become more complex, exhibiting additional nodes and antinodes. These higher modes correspond to higher natural frequencies and show more intricate deformation patterns across the plate, as illustrated in Fig. 3.

2.3 Strain Plot

Taking the second derivative of the displacement eigenfunction with respect to the spatial coordinates yields the strain equation (curvature eigenfunction). The displacement eigenfunction for a mode shape of a rectangular plate with all sides simply supported is typically expressed as:

$$W_{(x,y)} = \sin\frac{m\,\pi\,x}{a}\,.\,\sin\frac{n\,\pi\,y}{b} \qquad (m,n\ = 1,2,3,4\ldots\ldots\ldots..) \qquad \text{(from Eq. 19)}$$

The strain components, which are related to the curvature of the plate, can be derived by taking the second derivatives of the displacement eigenfunction with respect to x and y:

$$\frac{dw}{dx} = \left(\frac{m\,\pi}{a}\right).\sin\left(\frac{n\,\pi\,y}{b}\right).\ \cos\left(\frac{m\,\pi\,x}{a}\right). \qquad (21)$$

$$\frac{d^2w}{dx^2} = -\left(\frac{m\,\pi}{a}\right)^2.\ \sin\left(\frac{n\,\pi\,y}{b}\right).\ \sin\left(\frac{m\,\pi\,x}{a}\right). \qquad (22)$$

$$\frac{dw}{dy} = \left(\frac{n\,\pi}{b}\right).\sin\left(\frac{m\,\pi\,x}{a}\right).\ \cos\left(\frac{n\,\pi\,y}{b}\right). \qquad (23)$$

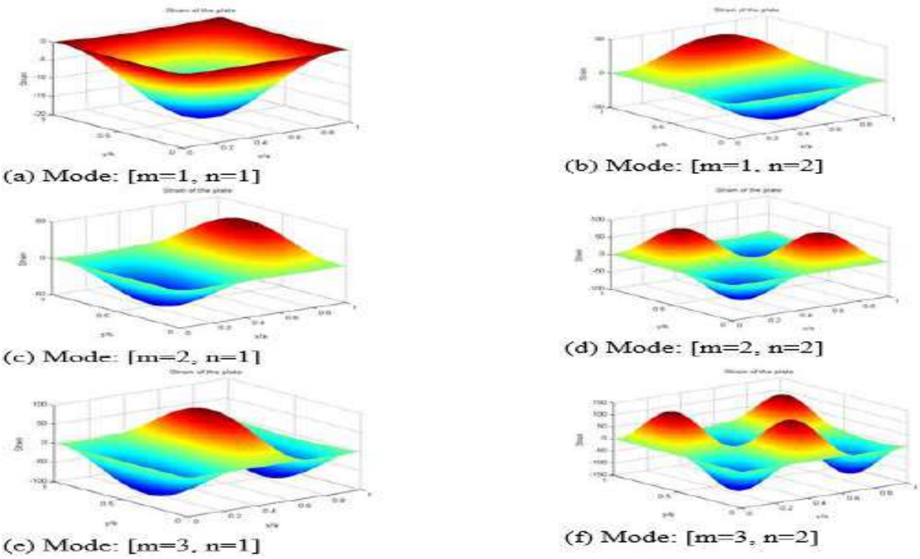
$$\frac{d^2w}{dy^2} = -\left(\frac{m\,\pi}{a}\right)^2.\ \sin\left(\frac{m\,\pi\,x}{a}\right).\ \sin\left(\frac{n\,\pi\,y}{b}\right). \qquad (24)$$

The strain equation for the plate, considering the curvature in both directions, is then:

$$\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} = \left[-\left(\frac{m\,\pi}{a}\right)^2.\sin\left(\frac{n\,\pi\,y}{b}\right).\sin\left(\frac{m\,\pi\,x}{a}\right)\right] + \left[-\left(\frac{m\,\pi}{a}\right)^2.\sin\left(\frac{m\,\pi\,x}{a}\right).\sin\left(\frac{n\,\pi\,y}{b}\right)\right]. \qquad (25)$$

$$\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} = -(\pi^2)\ \left(\sin\frac{m\,\pi\,x}{a}\right).\left(\sin\frac{n\,\pi\,y}{b}\right).\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]. \qquad (26)$$

Equation 26 describes the strain distribution in a rectangular plate with all edges simply supported for a given mode shape defined by the mode numbers m and n. The strain distribution reveals how the plate deforms under various vibrational modes, highlighting regions with maximum strain. This information is crucial for optimizing the placement of piezoelectric actuators and sensors. The strain distribution is plotted in MATLAB©, illustrating the maximum and minimum strain regions for various mode shapes. These plots are essential for visualizing the strain behavior of the plate and determining the optimal placement of piezopatches for effective active vibration control. Figure 4 shows the resulting strain plots.



(a) Mode: [m=1, n=1]
(b) Mode: [m=1, n=2]
(c) Mode: [m=2, n=1]
(d) Mode: [m=2, n=2]
(e) Mode: [m=3, n=1]
(f) Mode: [m=3, n=2]

Fig. 4 Strain plot for a simply supported plate

2.4 Objective Function

The curvature eigenfunction Eq. (26) for all the modes at different values of m and n can be algebraically summed to represent the overall strain distribution in the plate-type structure. Eq. (27), the combined strain equation, is used for a rectangular plate with all sides simply supported.

$W_{(x,y)}'' = \frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} = -(\pi^2)\left(\sin\frac{m\,\pi\,x}{a}\right).\left(\sin\frac{n\,\pi\,y}{b}\right).\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right].$	
$W'' = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{(x,y)}''.$	(27)

The combined strain equation, with M and N representing total modes in the X and Y directions, outlines the cumulative strain distribution across the plate. By algebraically summing the strain functions for different mode numbers m and n, the resulting equation gives a comprehensive view of the regions experiencing maximum strain. This combined strain equation (Eq. 27) is plotted in MATLAB©, as shown in Figure 5.

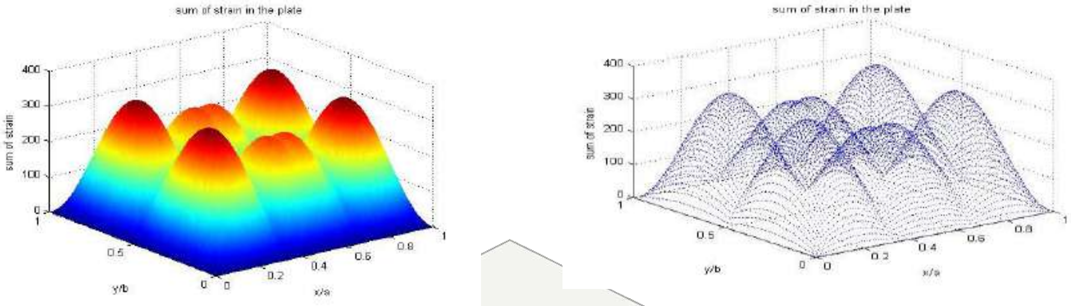


Fig. 5 Sum of strain plot

The Genetic Algorithm (GA) uses this equation as the objective function to determine optimal piezoelectric patch placements on a simply supported rectangular plate. The goal is to identify points of maximum strain where the piezoelectric patches can most effectively mitigate vibrations. By strategically placing the patches at these high-strain locations, the GA enhances the efficiency and effectiveness of the active vibration control system. The strain-based optimization positions piezoelectric patches to enhance vibration control, improving active system performance.

2.5 Optimal Position of Piezo Patches using GA

This study develops a binary-coded genetic algorithm (GA) to identify the optimal locations for piezoelectric actuators and sensors on a plate, focusing on controlling multimode vibrations. The GA's objective function is based on locating areas of maximum strain, making the design variables the coordinates along the plate's length (x) and width (y). Strain varies across these dimensions, guiding the algorithm to find the most effective actuator and sensor placements. The optimization process is subject to several constraints:

1. **Positional Constraints**: The optimal positions must lie within the plate's boundaries, defined by length (a) and width (b).
2. **Non-Overlapping Constraint**: When determining multiple actuator/sensor locations, the patches must not overlap.
3. **Variable Bound**: The x and y positions along the plate range from 0 to 1, representing normalized coordinates within the plate’s dimensions.

In scenarios requiring rapid vibration damping, a higher control force is needed. However, a single actuator has a voltage limit beyond which it may lose its properties. Multiple patches are used to meet the demand without exceeding the voltage limit. This study considers ten pairs of actuators and sensors to control the initial modes of vibration effectively. The GA optimizes the placement of these patches based on maximum strain locations, ensuring that the vibration control is both efficient and effective. The binary-coded GA employs the following operators and parameters:

- **Selection Operator:** Roulette wheel selection.
- **Crossover Operator:** Single-point crossover with an 80% probability.

- **Mutation Operator:** Bitwise mutation with a 5% probability.
- **Generations:** The GA runs for 90 generations to converge on an optimal solution.

The GA was implemented in MATLAB© by following these steps [34].

1. **Initialization:** Generate an initial population of potential solutions, each represented by binary strings encoding the x and y coordinates.
2. **Evaluation:** Compute the fitness of each solution based on the strain objective function.
3. **Selection:** The roulette wheel technique to select reproduction solutions, giving preference to those with higher fitness levels.
4. **Crossover:** Perform single-point crossover on selected solutions to generate offspring.
5. **Mutation:** Apply bitwise mutation to introduce variability.
6. **Iteration:** Repeat the evaluation, selection, crossover, and mutation steps for 90 generations.
7. **Final Solution:** Extract the best-performing solutions representing the optimal patch locations on the plate.

This approach strategically positions the piezoelectric patches to maximize vibration control efficiency, enhancing the overall performance of the active control system.

3 Results and Discussion

This research presents a simplified methodology for determining the optimal locations of piezoelectric patches on plate-type structures to control multiple vibration modes. The methodology leverages the displacement eigenfunction of the plate to trace the strain profile's maxima along its length and width. The Genetic Algorithm (GA) identifies the optimal x and y coordinates for placing piezoelectric patches at maximum strain values on the plate. The proposed algorithm uses the displacement eigenfunction to identify these points of maximum strain systematically. The GA efficiently searches the design space, converging on optimal locations with the highest strain for effective vibration control. The resulting x and y coordinates, corresponding to the positions of maximum strain identified by the GA, are presented below:

patch_position_x = 0.1900 0.8100 0.1900 0.8100 0.1900 0.8100 0.1900 0.8100 0.7900 0.7900

patch_position_y = 0.2800 0.2800 0.7200 0.7200 0.6900 0.6900 0.3100 0.3100 0.2800 0.7200

These coordinates represent the optimal locations for the piezoelectric patches. Placing the patches at these positions targets the critical areas of the plate with the highest strain. This maximizes the vibration control performance of the system.

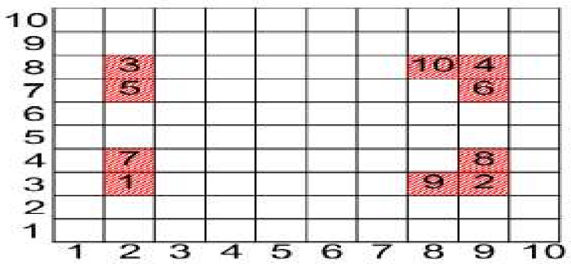


Fig. 6 Piezo patch location on plate structure

The figure marks the x and y positions of the identified optimal patch locations on the plate in Figure 6. This figure visually illustrates the strategic placement of the piezoelectric patches based on the GA results, highlighting the areas where maximum strain occurs. The optimal placement positions the patches to effectively counteract vibrations, enhancing the plate's overall dynamic performance.

The results are validated using another methodology widely adopted by researchers, which utilizes collocated piezoelectric patches. This approach identifies the optimal locations based on the highest position sensitivity of each vibration mode, focusing on controllability and observability. Many studies simultaneously optimize the placement of piezoelectric actuators and sensors to ensure the system is controllable and observable. This integrated approach maximizes control system performance using a cost function based on controllability and observability measures. In practical applications, engineers consider a finite number of vibration modes. Further result in spillover effects, where uncontrolled higher modes are unintentionally triggered. Such effects can degrade control performance, particularly in cases where higher frequency modes introduce instability or compromise system effectiveness. Hence, optimizing the placement of piezoelectric patches to minimize spillover is critical. Optimization techniques must account for these residual modes to ensure that higher-order modes do not adversely affect the control system. In the context of optimal piezo patch placement, we can formulate the equations for controllability and observability as follows:

3.1 Controllability Gramian Wc

The controllability Gramian Wc is a matrix that quantifies how easily a system can be controlled from the input space (actuators). It is defined as:

$$W_c = \int_0^{\infty} e^{At}\, B\, B^T\, e^{A^T t}\, dt. \quad (28)$$

Where A is the system matrix (defining the system's dynamics), B is the input matrix (defining how the input affects the system). The goal of actuator placement optimization is to maximize the trace or determinant of Wc, indicating enhanced controllability:

$$\text{Maximize} \quad \text{trace}(W_c) \text{ or } \det(W_c)$$

3.2 Observability Gramian Wo

The observability Gramian Wo is a matrix that measures how well the internal states of a system can be inferred from the outputs (sensors). It is defined as:

$$W_o = \int_0^{\infty} e^{A^T t}\, C^T C e^{At}\, dt. \quad (29)$$

The system matrix, A, represents the system dynamics, while the output matrix, C, maps the system's states to the output. The goal of sensor placement optimization is to maximize the trace or determinant of Wo, indicating enhanced observability:

$$\text{Maximize} \quad \text{trace}(W_o) \text{ or } \det(W_o)$$

3.3 Combined Optimization Objective:

When optimizing actuator and sensor placement simultaneously, a common approach is defining a combined objective function that incorporates controllability and observability Gramians. One possible objective function is:

$$J = \alpha.\text{trace}(W_c) + \beta.\text{trace}(W_o). \quad (30)$$

Where, α and β are weighting factors that balance the importance of controllability and observability. The optimization problem can then be formulated as:

$$\text{maximize } J$$

Subject to the constraints on the physical placement of piezo patches on the structure. For a simply supported rectangular plate, the controllability and observability equations incorporate boundary-specific mode shapes.

The controllability and observability Gramians for this case can be expressed.

3.4 Controllability Gramian Wc

The mode shapes can be used to define the controllability Gramian Wc for a plate whose edges are all simply supported.

$$W_c = \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{B_{mn} B_{mn}^{\;T}}{\lambda_{mn}}. \quad (31)$$

Where: B_{mn} is the mode participation factor for the actuator placed at a specific location, λ_{mn} is the eigenvalue corresponding to the (m, n).

3.5 Observability Gramian Wo

Similarly, the observability Gramian Wo is given by:

$$W_o = \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{C_{mn}^{\;T} C_{mn}}{\lambda_{mn}}. \quad (32)$$

Where C_{mn} is the mode participation factor for the sensor placed at a specific location, λ_{mn} is the eigenvalue corresponding to the (m, n) mode.

3.6 Combined Optimization Objective

The combined objective function for the optimal placement of piezoelectric patches (actuators and sensors) can be expressed as:

$$J = \alpha \cdot \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{B_{mn} B_{mn}^{T}}{\lambda_{mn}} + \beta \cdot \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{C_{mn}^{T} C_{mn}}{\lambda_{mn}}. \quad (33)$$

Here, the goal is to maximize J with respect to the locations of the piezoelectric patches on the plate, ensuring optimal controllability and observability. The piezo patch placement must adhere to the plate's physical constraints, ensuring they are within its boundaries. The patches should be positioned to maximize the controllability and observability criteria. In the controllability and observability Gramians, B_{mn} and C_{mn} represent the mode participation factors for the actuator and sensor, respectively, for a given mode (m, n) of the plate. These factors quantify how much a particular mode is influenced by the actuator or sensed by the sensor. The mode participation factor describes how effectively the actuator excites the (m, n) mode when placed at a particular location on the plate. It can be stated mathematically as,

$$B_{mn} = \int_0^a \int_0^b W_{xy} \emptyset_{A_{xy}} \, dxdy, \quad \text{and} \quad (34)$$
$$C_{mn} = \int_0^a \int_0^b W_{xy} \emptyset_{S_{xy}} \, dxdy.$$

Where, W_{xy} is mode shape, $\emptyset_{A_{xy}}$ and $\emptyset_{S_{xy}}$ are the shape function of the actuator, and sensor's influence. The **controllability** and **observability** of the simply supported plate on all sides, considering the spillover effect, are analyzed and plotted using MATLAB©. These plots are shown in **Figure 7.** The spillover effect, caused by higher-order modes not directly controlled, is accounted for to maintain effective control and sensing without exciting unintended modes.

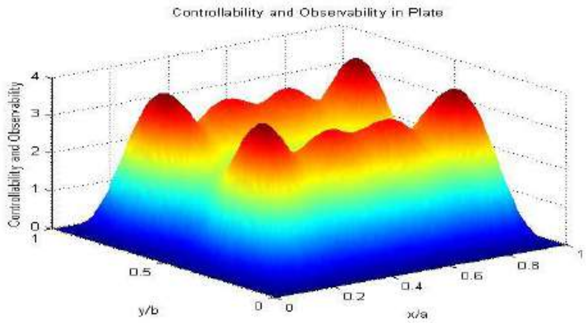


Fig. 7 Controllability and observability plot

This study compares the patch positions identified with those determined by other widely used methodologies. Researchers such as Bruant et al. [35], Narwal and Chhabra [36], and Ning [37] have used these methodologies to locate the optimal positions of piezoelectric patches based on controllability and observability. Both methodologies identify identical patch positions for different modes, as listed in the Table 1.

Table 1 Piezoelectric patch position with two different methodologies.

Mode (m, n)	Developed Methodology				Controllability Observability			
	First	Second	Third	Fourth	First	Second	Third	Fourth
1. m=1, n=1	6, 6	6, 5	5, 6	5, 5	6, 6	6, 5	5, 6	5, 5
2. m=2, n=1	3, 6	8, 6	3, 5	8, 5	3, 6	8, 6	3, 5	8, 5
3. m=1, n=2	6, 3	6, 8	5, 3	5, 8	6, 3	6, 8	5, 3	5, 8
4. m=3, n=1	6, 6	9, 6	6, 5	2, 6	6, 6	9, 6	6, 5	2, 6
5. m=2, n=2	3, 3	8, 3	3, 8	8, 8	3, 3	8, 3	3, 8	8, 8
6. m=3, n=2	6, 3	6, 8	9, 3	9, 8	6, 3	6, 8	9, 3	9, 8

The optimal locations of the piezoelectric patches, determined through a GA using the sum of strain as the objective function, are shown in Fig. 6. These positions precisely match the optimal locations derived from maximizing the controllability and observability of the simply supported plate, as illustrated in Fig. 8. Both approaches yield the same patch positions, as highlighted in Fig. 6.

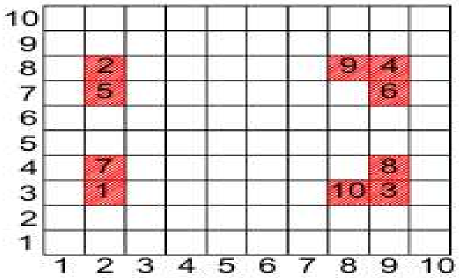


Fig. 8 Piezo patch location on plate structure

The results show a close alignment when compared with the proposed methodology, which uses the displacement eigenfunction and strain profile to determine patch placement. Both approaches identify critical high-strain regions on the plate, though the proposed methodology simplifies the process by reducing computational complexity. The comparison shows that the controllability and observability method focuses more directly on system performance. In contrast, the proposed strain-based approach offers a computationally efficient path to optimal placement with similar effectiveness. This validation reinforces the practical applicability of the proposed methodology in real-world scenarios, ensuring that the optimized placement is robust across different criteria for performance maximization.

The proposed methodology successfully identifies the optimal piezoelectric patch locations, providing a robust solution for controlling multimode vibrations in plate-type structures. Using the displacement eigenfunction and the GA demonstrates a powerful approach to optimizing the design of active vibration control systems.

4. Conclusion

This paper introduces a streamlined approach for determining the optimal placement of piezoelectric patches on plate-type structures to enhance multimode vibration control. The methodology uses the displacement eigenfunction and a GA to identify areas with maximum strain on the plate. Further facilitating the strategic placement of piezoelectric actuators and sensors. The study examines explicitly a pinned rectangular plate at all the edges, using its strain profile as the objective function for the optimization process. The results demonstrate that the proposed methodology can efficiently determine optimal patch locations, enhancing vibration control performance. Positioning multiple piezoelectric patches at optimal locations enhances vibration damping across multiple modes. This effectively minimizes overall structural vibrations.

The methodology developed in this study provides an efficient solution for optimizing piezoelectric patch placement on flexible structures. It is particularly effective in complex multimode vibration scenarios. Leveraging strain distribution and GA optimization simplifies identifying optimal actuator and sensor locations, enhancing control system performance. This work advances active vibration control technology, offering a robust framework to enhance the dynamic performance and stability of plate-type structures. It holds potential applications across various engineering fields.

Future work could expand the methodology to account for different boundary conditions, such as clamped-clamped, clamped-free, or simply supported-free, increasing its versatility. Further research may focus on optimizing irregularly shaped plates or plates with varying thicknesses, making the approach more applicable to complex structures. Additionally, the methodology could be refined to handle dynamic boundary conditions and real-time adaptation in response to operational changes. This study is limited to simulations of regularly shaped, uniformly thick flexible plates, excluding irregular geometries or variable thicknesses. It also focuses solely on simply supported boundary conditions, without considering combinations of other conditions like clamped, hinged, or free edges.

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